

Dimensional Analysis, Similitude and Modeling:

Simple method of predicting physical phenomena.

For instance, $\underline{F_D} = f(\underline{l}, \underline{V}, \underline{\rho}, \underline{g})$

Collect these 5 parameters into a smaller number non-dimensional terms, so that I can conduct my experiments faster and cheaper.

Dimensionless groups are called the Π terms.

if I have "N" number of Π terms

$$\Pi_N = f(\Pi_1, \Pi_2, \Pi_3, \dots, \Pi_{N-1})$$

General Dimensions:

in fluid mechanics, M-L-T or F-L-T is sufficient to represent flow properties.

$$g = \frac{m}{V} = \frac{[M]}{[L]^3} = [ML^{-3}]$$

$$g = \frac{m}{v} \quad F = m \cdot a \Rightarrow [F] = [M][L T^{-2}]$$

$$[M] = [F][L^{-1} T^2]$$
$$g = \frac{[F L^{-1} T^2]}{[L^3]} = [F L^{-4} T^2]$$

↑ ↑
Dimensionally homogeneous

Buckingham Π theorem:

If a flow phenomenon depends on k physical parameters that involve R basic dimensions (M, L, T or F, L, T) then there exists only $k - R$ independent Π terms that can be formed from these physical parameters.

Steps: 1) Identify k parameters

2) Express k parameters in terms of M, L, T or (F, L, T)

3) Identify R value

4) Select R out of k parameters

* combined R parameters must contain all basic dimensions

* each parameter must be dimensionally independent.

5) Select one additional parameter and use $(R+1)$ parameters to form Π terms

6) Repeat step 5 $(k-R-1)$ times.

Example: Consider the drag force (F_D) on an object with a characteristic length of (l), moving slowly at velocity (v) through air. The drag force also depends on (μ, ρ)

$$1) F_D = f(l, v, \mu, \rho)$$

$$2) F_D = [MLT^{-2}]$$

$$l = [L]$$

$$v = [LT^{-1}]$$

$$\mu = [ML^{-1}T^{-1}]$$

$$\rho = [ML^{-3}]$$

$$\tau = \mu \frac{du}{dy}$$

$$[ML^{-1}T^{-2}] = \left[\frac{ML}{L^2} \right] \left[\frac{L^2}{L^2} \right]$$

$$3) R=3 \quad (M, L, T)$$

$$4) \text{ select 3 parameters. } (\underline{g}, \underline{l}, \underline{v})$$

$$5) \Pi_1 \rightarrow (F_0, g, l, v)$$

$$6) \Pi_2 \rightarrow (\nu, g, l, v)$$

$$\Pi_1 \rightarrow F_0 \cdot g^a \cdot l^b \cdot v^c$$

$$\rightarrow [M L T^{-2}] [M L^{-3}]^a [L]^b [L T^{-1}]^c$$

$$M^0 L^0 T^0 \rightarrow M^{1+a} L^{1+(-3a)+b+c} T^{-2-c}$$

$$1+a=0 \quad 1-3a+b+c=0 \quad -2-c=0$$

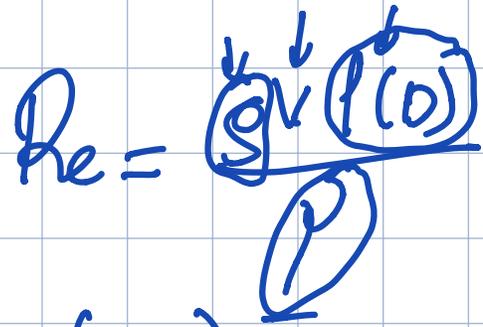
$$\underline{a=-1} \quad 1+3+b-2=0 \quad \underline{c=-2}$$

$$\underline{b=-2}$$

$$\Pi_1 = F_0 \cdot g^{-1} \cdot l^{-2} \cdot v^{-2}$$

$$\boxed{\Pi_1 = \frac{F_0}{g l^2 \cdot v^2}} = \frac{[M L T^{-2}]}{[M L^{-3}] [L]^2 [L T^{-1}]^2}$$

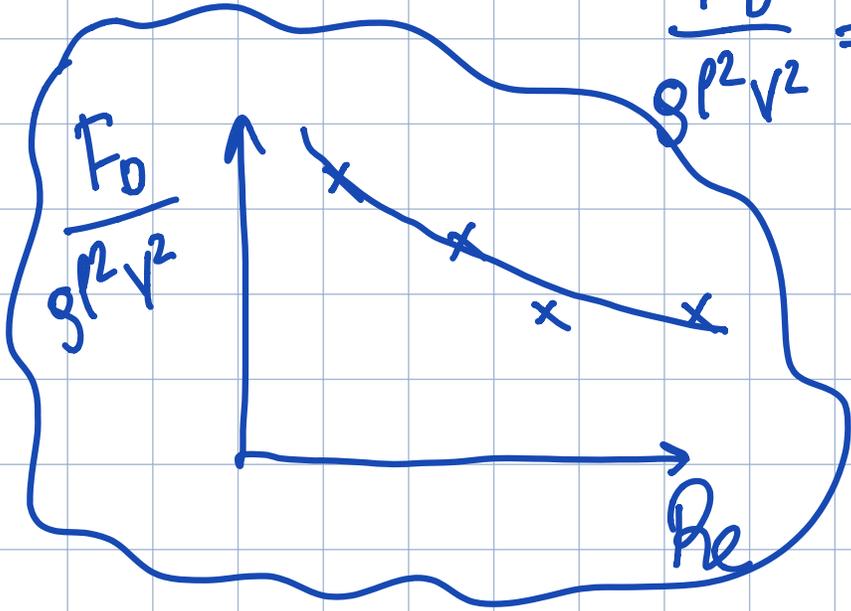
$$\Pi_2 = (\underline{\rho}, \underline{g}, l, \underline{v})$$



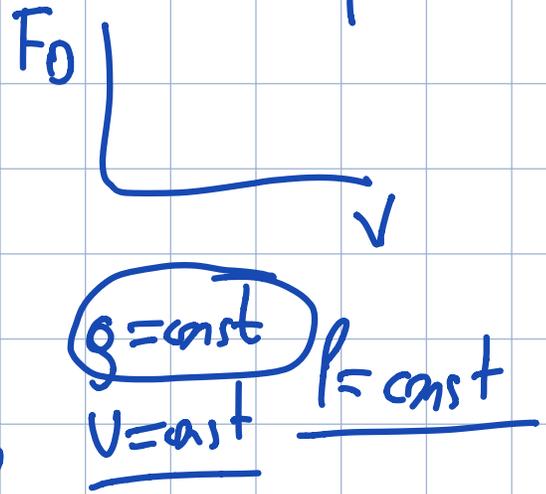
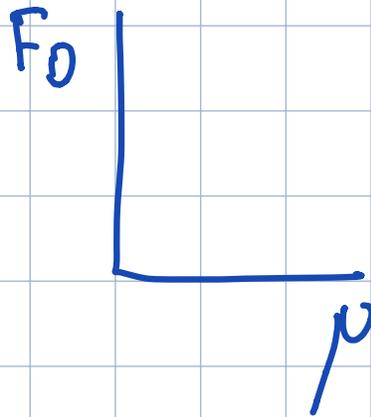
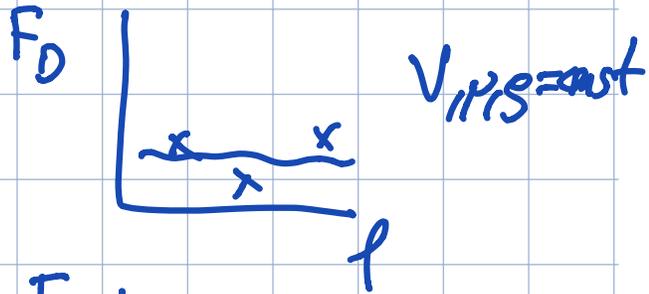
$$\Pi_2 = Re$$

$$\Pi_1 = f_{xn}(\Pi_2)$$

$$\frac{F_0}{g l^2 v^2} = f_{xn}(Re)$$



$$F_0 = f(\rho, v, \mu, g)$$



Common Non-Dimensional Numbers in Fluid Mechanics:

$$\text{Reynolds Number} = \frac{\rho V \cdot l (D_H)}{\mu} = \frac{\text{inertia force}}{\text{viscous force}}$$

$$\text{Euler Number} : \frac{\Delta p}{\rho V^2} = \frac{\text{pressure force}}{\text{inertia force}}$$

$$\text{Froude Number} : \frac{V}{\sqrt{g l}} = \frac{\text{inertia force}}{\text{gravitational force}}$$

$$\text{Weber Number} : \frac{\rho V^2 l}{\sigma} = \frac{\text{inertia force}}{\text{surface tension force}}$$

$$\text{Mach Number} : \frac{V}{c} = \frac{\text{inertia force}}{\text{compressibility force}}$$

$$\text{Strouhal Number} : \frac{w \cdot l}{V} = \frac{\text{local inertia force}}{\text{convective inertia force}}$$

$$\text{Drag Coefficient} : \frac{F_D}{\frac{1}{2} \rho V^2 l^2} = \frac{\text{drag force}}{\text{inertia}}$$

$$\text{Lift Coefficient} : \frac{F_L}{\frac{1}{2} \rho V^2 l^2} = \frac{\text{lift force}}{\text{inertia}}$$

For a turbulent flow in a pipe, the pressure drop (Δp) is a function of the pipe diameter, (D), velocity of the flow (V), the length of the pipe (L), the fluid density (ρ) and viscosity (μ), the relative roughness. (ϵ/D). Find a suitable set of Π terms

$$1) \Delta p = f(D, V, L, \rho, \mu, \epsilon/D)$$

$$2) \Delta p = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$

$$D = [L]; V = [LT^{-1}]; L = [L]$$

$$\rho = [ML^{-3}]; \mu = [ML^{-1}T^{-1}]; \frac{\epsilon}{D} \text{ unitless}$$

$$3) R = 3$$

$$4) \left(\underline{D}, \underline{V}, \underline{L} \right) \quad \left(\underline{D}, \underline{V}, \underline{\rho} \right)$$

$$5) \Pi_1 (\Delta p, D, V, \rho) \rightarrow \frac{\Delta p}{\rho V^2} = \frac{ML^{-1}T^{-2}}{[ML^{-3}][LT^{-1}]^2}$$

$$6) \Pi_2 (L, D, V, \rho) \rightarrow \frac{L}{D}$$

$$\Pi_2 (L, D, V, \rho) \rightarrow \frac{L}{D} \quad \rho V D$$

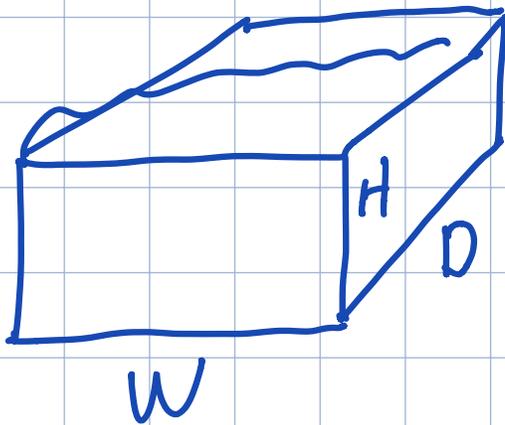
$$\Pi_3 (\mu, \nu, \nu, g) \rightarrow Re = \frac{\rho V L}{\mu}$$

$$\Pi_4 (\epsilon/D, D, V, g) \rightarrow \epsilon/D$$

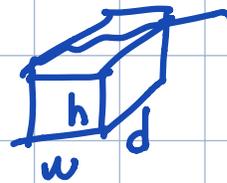
$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4) \quad \frac{\Delta P}{\rho V^2} = f\left(\frac{L}{D}, \frac{\epsilon}{D}, Re\right)$$

Similarity and Modeling:

Geometric Similarity:



$$\frac{B}{b} = \frac{H}{h} = \frac{W}{w}$$



Kinematic Similarity: velocities/acceleration

Dynamic Similarity: force

Modeling: Predict forces (lift, drag) exerted by the flow on cars/wings/ships.

Flow around the full-scale body: prototype

$$\Pi_1^{(p)} = f(\Pi_2^{(p)}, \Pi_3^{(p)}, \dots, \Pi_{N-1}^{(p)})$$

test a scaled down model in a wind tunnel

$$\Pi_1^{(m)} = f(\Pi_2^{(m)}, \Pi_3^{(m)}, \dots, \Pi_{N-1}^{(m)})$$

$$\Pi_1^{(p)} = \Pi_1^{(m)} \rightarrow \text{prediction equation}$$

$$\Pi_2^{(p)} = \Pi_2^{(m)}$$

$$\Pi_3^{(p)} = \Pi_3^{(m)}$$

$$\vdots$$

$$\Pi_{N-1}^{(p)} = \Pi_{N-1}^{(m)}$$

} similarity requirements

A $\frac{1}{5}$ scale of a vehicle is to be tested in two tunnels. One in a wind tunnel, second in a water tunnel. Determine the maximum velocity of water and air in the tunnels if the maximum speed of the prototype is 120 mph.

$$Re = \frac{\rho V l}{\mu} \quad V = \frac{\nu}{g} \quad Re = \frac{V \cdot l}{\nu}$$

$$Re^{(m)} = Re^{(p)} \rightarrow \text{air}$$

$$\frac{V_m \cdot l_m}{\nu_m} = \frac{V_p \cdot l_p}{\nu_p}$$

$$V_m = V_p \cdot \left(\frac{l_p}{l_m} \right) \cdot \left(\frac{\nu_m}{\nu_p} \right) \Rightarrow \underline{V_m = 600 \text{ mph}}$$

\downarrow 120 \downarrow 5 \downarrow 1

$$Re^{(m)} = Re^{(p)} \rightarrow 1.21 \times 10^{-5} \text{ ft}^2/\text{s}$$

$$V_m = V_p \cdot \left(\frac{l_p}{l_m} \right) \cdot \left(\frac{\nu_m}{\nu_p} \right)$$

\downarrow 120 mph \downarrow 5 \downarrow $1.57 \times 10^{-4} \text{ ft}^2/\text{s}$

$$\underline{V_m = 46.2 \text{ mph}}$$

Example: A model with a scale of $\frac{1}{4}$ is to be tested to determine the velocity of discharge from a pinhole crack on the side of a pressured tank. The velocity is a function of the pressure in the tank, wall thickness, diameter of the pinhole crack and the viscosity of the fluid in the tank. Take the viscosity scale as $\frac{1}{4}$.

a) Find the prediction equation and similarity requirements

b) Find the velocity scale

$$1) \underline{V} = f(\underline{p}, \underline{l}, \underline{d}, \underline{\nu})$$

$$2) V = [LT^{-1}] \quad p = [FL^{-2}]$$

$$l = [L] \quad d = [L] \quad \nu = [FL^{-2}T]$$

$$\tau = \nu \cdot \frac{du}{dy} \quad [FL^{-2}] = [\nu] \cdot \frac{[KT^{-1}]}{[KT^{-1}]}$$

$$\nu = [FL^{-2}T]$$

$$3) R=3$$

$$4) \cancel{(V, l, d)} \quad (d, \nu, p) \checkmark$$

$$5) \Pi_1 \rightarrow (V^a, d^b, \nu^c, p^d) \quad \begin{matrix} a = -1 \\ b = 1 \\ c = -1 \end{matrix}$$

$$\Pi_2 \rightarrow (P, d, \rho, \rho) \rightarrow \frac{f}{d}$$

$$\Pi_1 = \frac{V \cdot \rho}{\rho \cdot d} \frac{[L T^{-1}] [F L^{-2} T^{-2}]}{[F L^{-2}] [L]} \checkmark$$

$$\frac{V \cdot \rho}{\rho \cdot d} = f\left(\frac{f}{d}\right)$$

$$\frac{V_m \rho_m}{\rho_m d_m} = \frac{V_p \rho_p}{\rho_p d_p} \quad \text{prediction equation}$$

$$\frac{\rho_m}{d_m} = \frac{\rho_p}{d_p} \quad \text{similarity requirement.}$$

$$\frac{V_m}{V_p} = \left(\frac{\rho_m}{\rho_p}\right) \left(\frac{d_m}{d_p}\right) \left(\frac{\rho_p}{\rho_m}\right)$$

$$\frac{d_m}{d_p} = \frac{1}{4}$$

$$\frac{\rho_m}{\rho_p} = 1.4$$

$$\boxed{\frac{V_m}{V_p} = \frac{1}{5.6}}$$

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