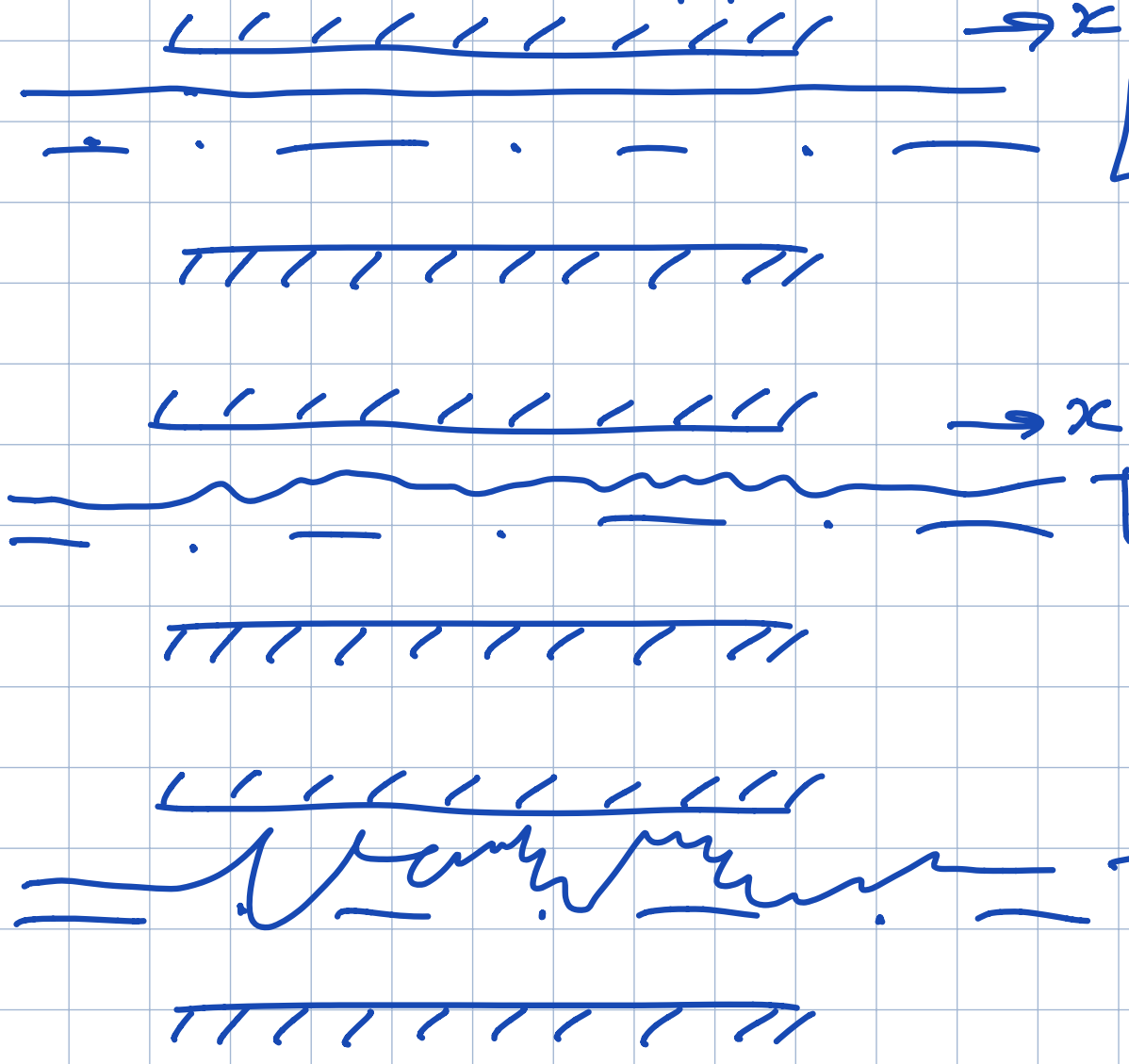
Viscous flow in pipes:



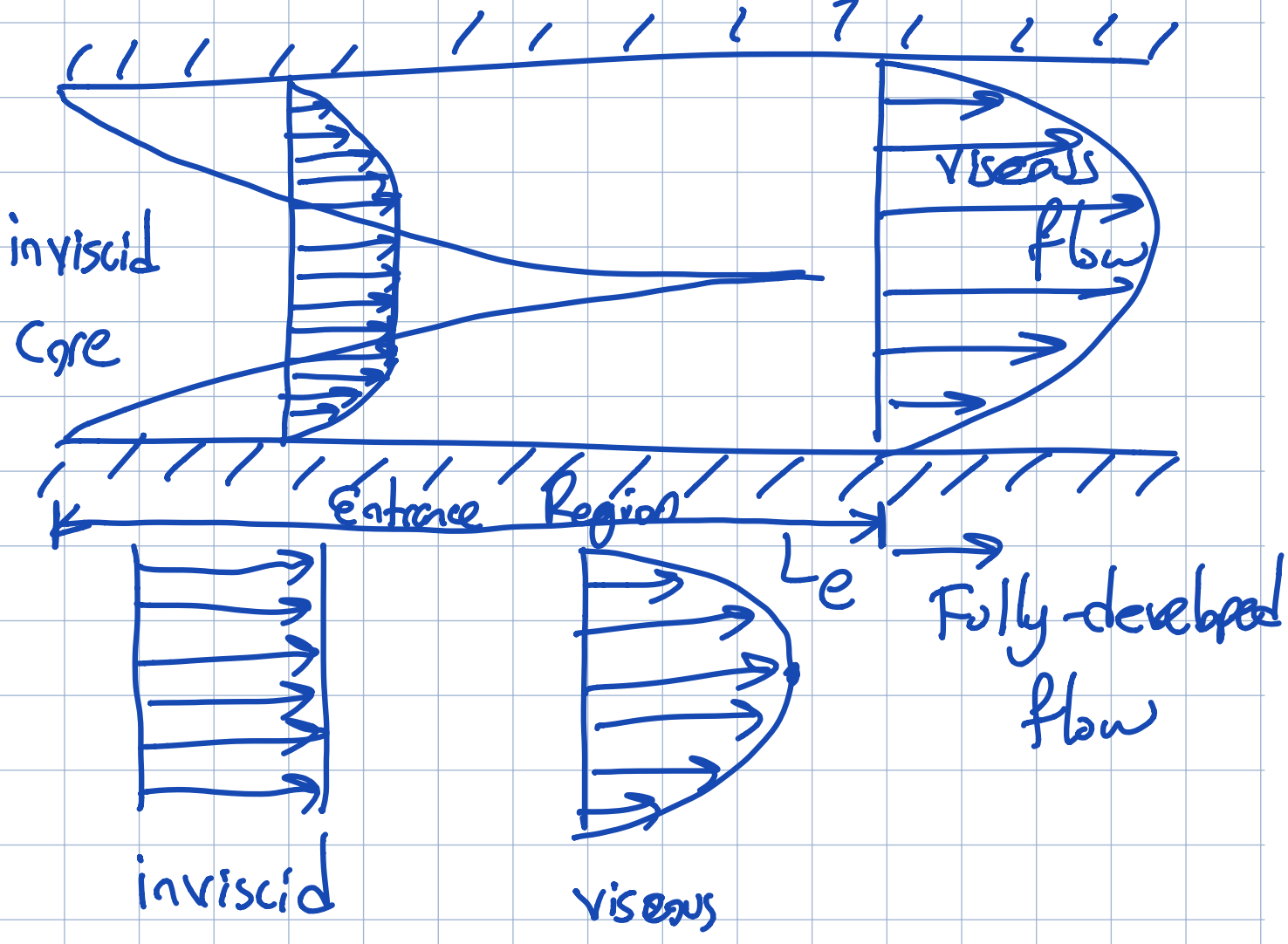
$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

$Re < 2000$ Laminar

$2000 < Re < 2500$ transitional

$Re > 2500$ turbulent

Entrance Region and Fully-developed flow

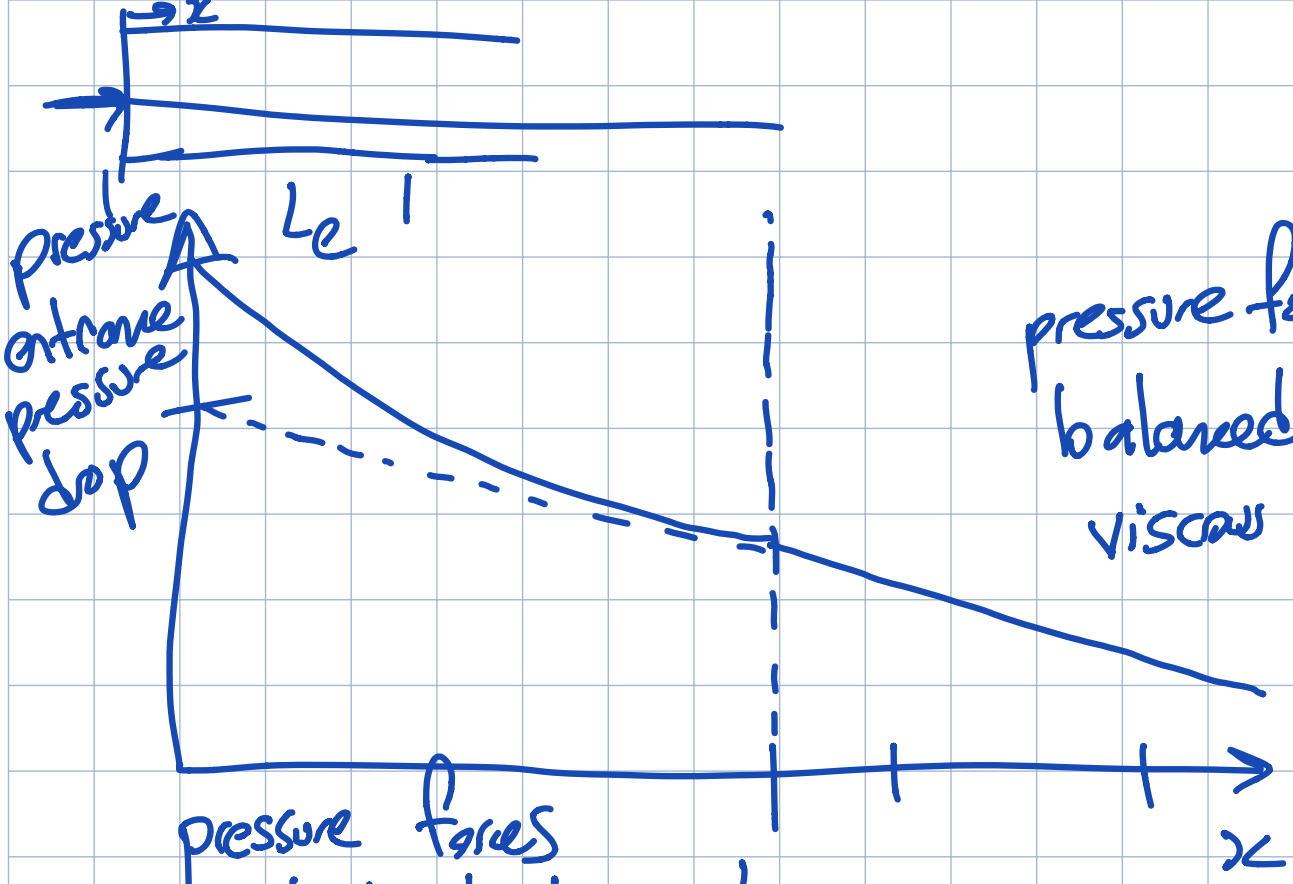


$$\frac{Le}{D} = 0.06\,Re \qquad \text{Laminar Flow}$$

$Re \uparrow_{2000}$

$$Le = 120\,D$$

$$\frac{Le}{D} = 4.4\,(Re)^{1/6} \qquad \text{Turbulent Flow}$$



$\frac{\partial p}{\partial x} \neq \text{const}$

entrance region

$\frac{\partial p}{\partial x} = \text{const}$

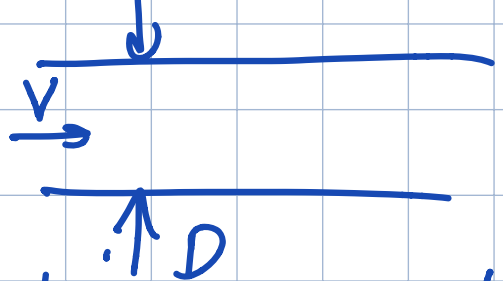
fully developed flow.

Fully-developed Laminar Flow:

from dimensional analysis

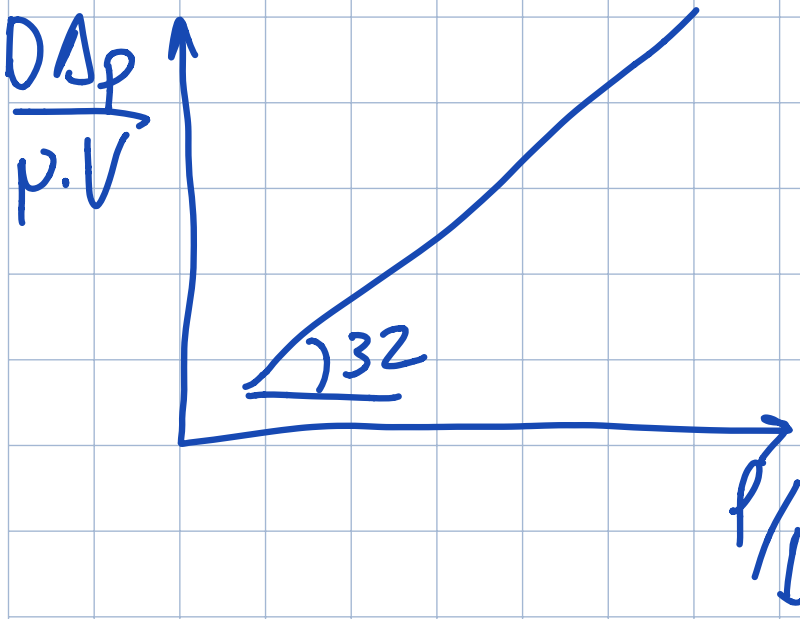
$\Delta p = f(V, \rho, D, \mu)$

$k=5 \quad R=3$



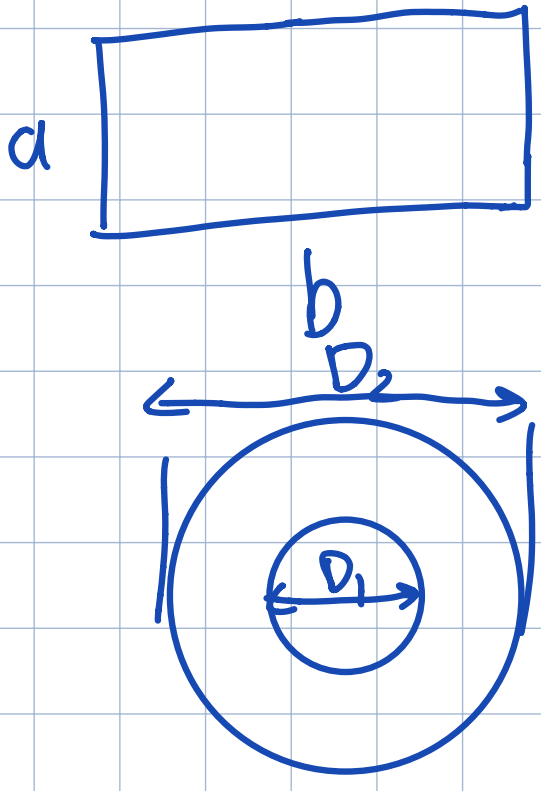
$$\frac{D.\Delta p}{\mu.V} = fn\left(\frac{l}{D}\right)$$

Π_1 Π_2



$$\left(\frac{D\Delta p}{\mu.V}\right) = C.\frac{l}{D}$$

32 for a pipe circular pipe



$\frac{a}{b}$	C
0.1	84.7
$\frac{D_1}{D_2}$	
0.1	89

$$Q = V.A$$

$$Q = \frac{\Delta p . D^2}{32 \mu . l} . \frac{\pi D^2}{4}$$

$$\boxed{Q = \frac{\pi D^4 \Delta p}{128 \mu . l}}$$

$$\frac{D \Delta p}{\mu . V} = 32 . \frac{l}{D}$$

$$V = \frac{\Delta p . D^2}{32 \mu . l}$$

$$\Delta p = \frac{32 . \mu . l . V}{D^2}$$

$$\frac{\Delta p}{\frac{1}{2} \rho V^2} = \frac{\frac{32 . \mu l . V}{D^2}}{\frac{1}{2} \rho V^2} = 64 . \underbrace{\left(\frac{\mu}{\rho V D} \right)}_{\frac{1}{Re}} \left(\frac{l}{D} \right)$$

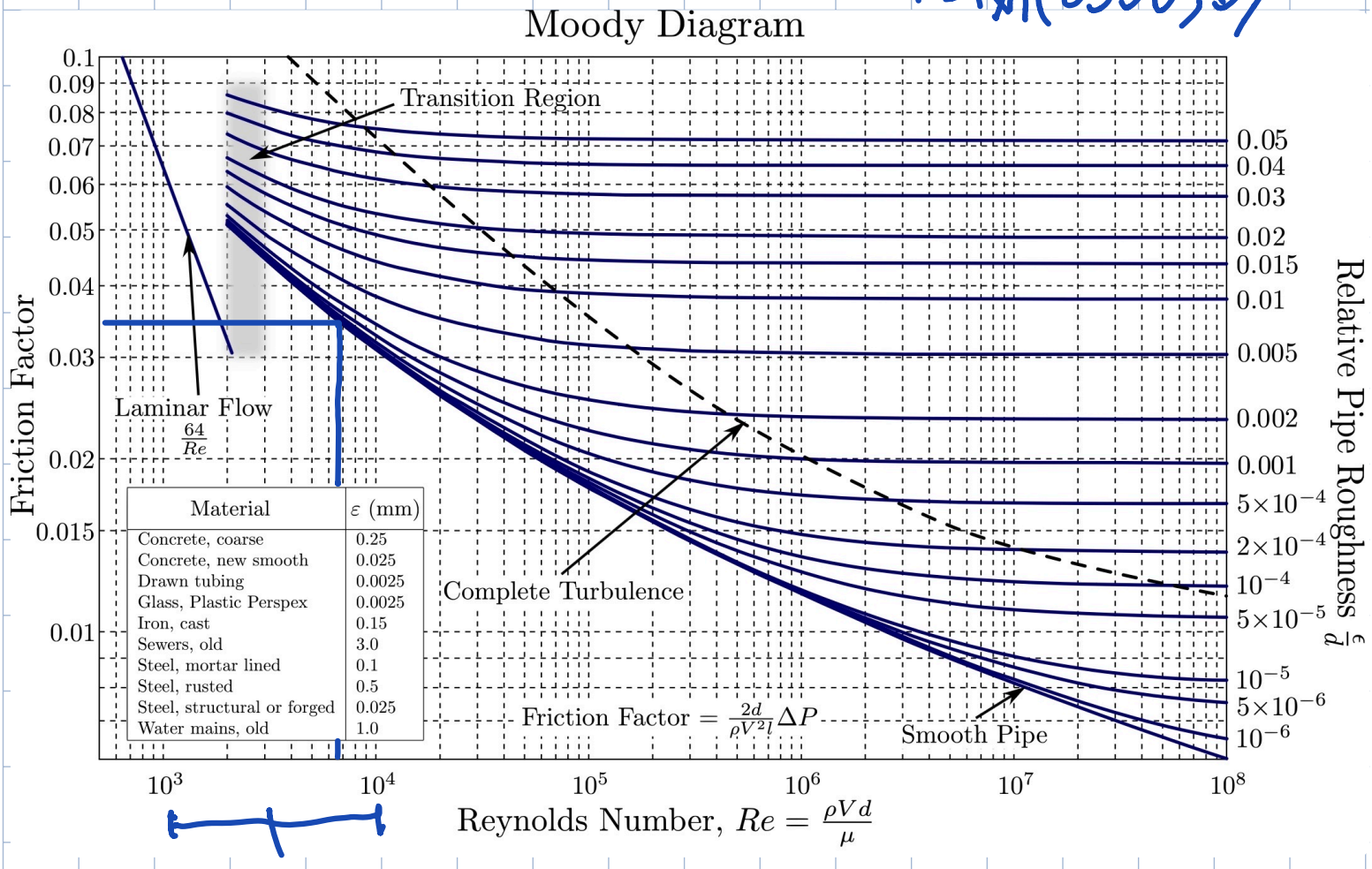
$$\Delta p = \left(\frac{64}{Re} \right) \left(\frac{l}{D} \right) \left(\frac{\rho V^2}{2} \right)$$

↳ f Darcy friction factor

$$f = \frac{64}{Re} \qquad \frac{Q}{}$$

$f = \frac{64}{Re}$ for Laminar Only

$f = f_{vn}(6350, 0)$



Material	ε (mm)
Concrete, coarse	0.25
Concrete, new smooth	0.025
Drawn tubing	0.0025
Glass, Plastic Perspex	0.0025
Iron, cast	0.15
Sewers, old	3.0
Steel, mortar lined	0.1
Steel, rusted	0.5
Steel, structural or forged	0.025
Water mains, old	1.0

Viscous flow in pipes – Turbulent flow:

Turbulent flow introduces significant complexity to our analysis.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

h_p: power input

h_L: head loss

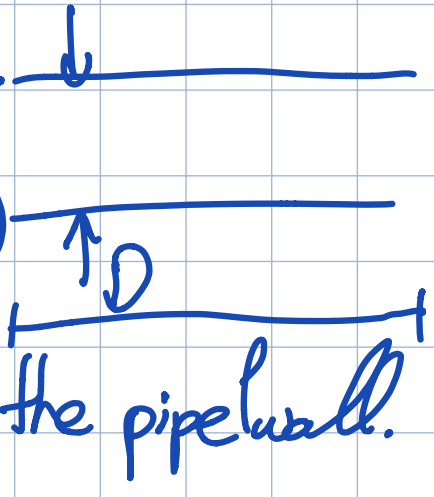
$h_L = h_{L\,major} + h_{L\,minor}$

$h_{L\,major}$ → due to viscous effects in straight pipes.

$h_{L\,minor}$ → due to various pipe components (valves, bends, tees)

Major losses:

$$\Delta p = f_{xn}(V, D, l, \mu, \rho, \varepsilon)$$



ε → measure of roughness of the pipe wall.

$k = 7 \quad R = 3 \Rightarrow 4\pi$

(D, V, ρ)

$\Pi_1 \rightarrow (\Delta p, D, \rho, V) \Rightarrow \dfrac{\Delta p}{\frac{1}{2}\rho V^2}$

$\Pi_2 \rightarrow (l, D, \rho, V) \Rightarrow l/D$

$\Pi_3 \rightarrow (\mu, D, \rho, V) \Rightarrow Re\left(\dfrac{\rho V D}{\mu}\right)$

$\Pi_4 \rightarrow (\varepsilon, D, \rho, V) \Rightarrow \dfrac{\varepsilon}{D}$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = f_{xn}\left(\frac{l}{D}, Re, \frac{\epsilon}{D}\right)$$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} \propto \frac{l}{D}$$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{l}{D} f_{xn}\left(Re, \frac{\epsilon}{D}\right)$$

Laminar $\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{l}{D} \cdot f$ $f \nearrow \frac{64}{Re}$

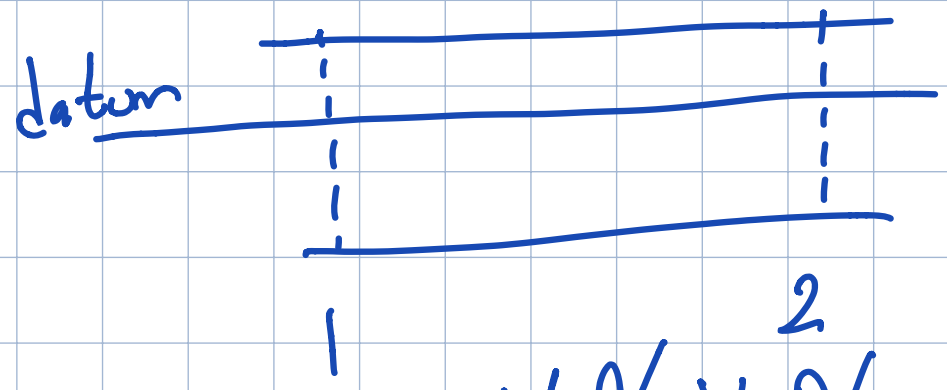
↳ Darcy friction factor

$f = f_{xn}\left(Re, \frac{\epsilon}{D}\right)$ turbulent

$f = \frac{64}{Re}$ Laminar

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\frac{P_1}{\gamma} + z_1 + h_p = \frac{P_2}{\gamma} + z_2 + h_L$$



$h_{L\,major}$

$z_1 = z_2$

$h_p = 0$

$h_{L\,minor} = 0$

$$V_1 A_1 = V_2 A_2$$

$$h_{L\,major} = \frac{P_1 - P_2}{\gamma} = \frac{\Delta P}{\gamma}$$

$$\Delta p = f \cdot \frac{l}{D} \cdot \frac{\rho V^2}{2}$$

$$h_{L\,major} = \frac{f \cdot \frac{l}{D} \cdot \frac{\rho V^2}{2}}{\rho g}$$

$$\boxed{h_{L\,major} = f \cdot \frac{l}{D} \cdot \frac{V^2}{2g}}$$

$$f = f\left(\frac{D}{\varepsilon}\right)$$

$$f = f\left(Re, \frac{\varepsilon}{D}\right)$$

	ε feet	ε mm
steel	0.003-0.03	0.9-9 mm
PVC	0	0
Drawn tubing	0.000005	0.0015

Colebrook formula

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Modified Colebrook Formula (Haaland formula)

$$\frac{1}{\sqrt{f}} = -1.8\log\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right)$$

Find the Darcy friction factor for a flow with a Reynolds number of 5000 in a 1cm diameter cast iron pipe by incorrectly treating the flow as laminar and

by treating the flow as turbulent?

$$f = \frac{64}{Re} \quad \text{Laminar}$$

$$f_{Laminar} = \frac{64}{5000} = \underline{0.0128}$$

$$f = f_n\left(Re, \frac{\varepsilon}{D}\right) \qquad f = f_n\left(5000, 0.025\right)$$

$$5000, \quad \frac{0.25\ mm}{10\ mm} \qquad \underline{f = 0.063}$$

Minor losses:

Loss coefficient: K_L

$$K_L = \frac{h_{L\,minor}}{V^2/2g} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

$$\boxed{h_{L\,minor} = K_L \cdot \frac{V^2}{2g}}$$

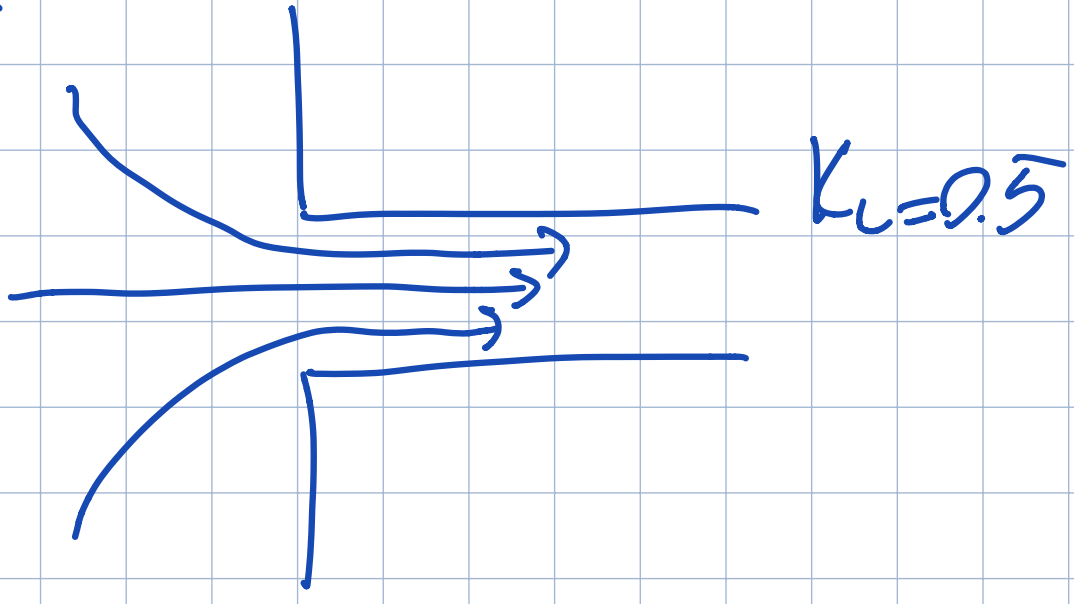
$2g$

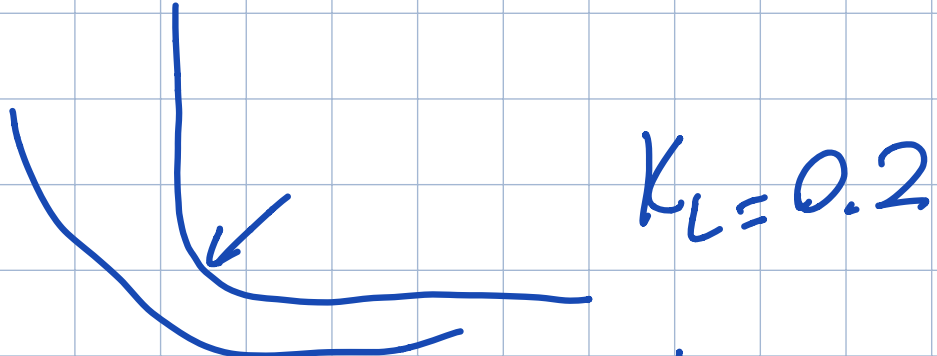
Actual value of K_L fxn(geometry, Re)

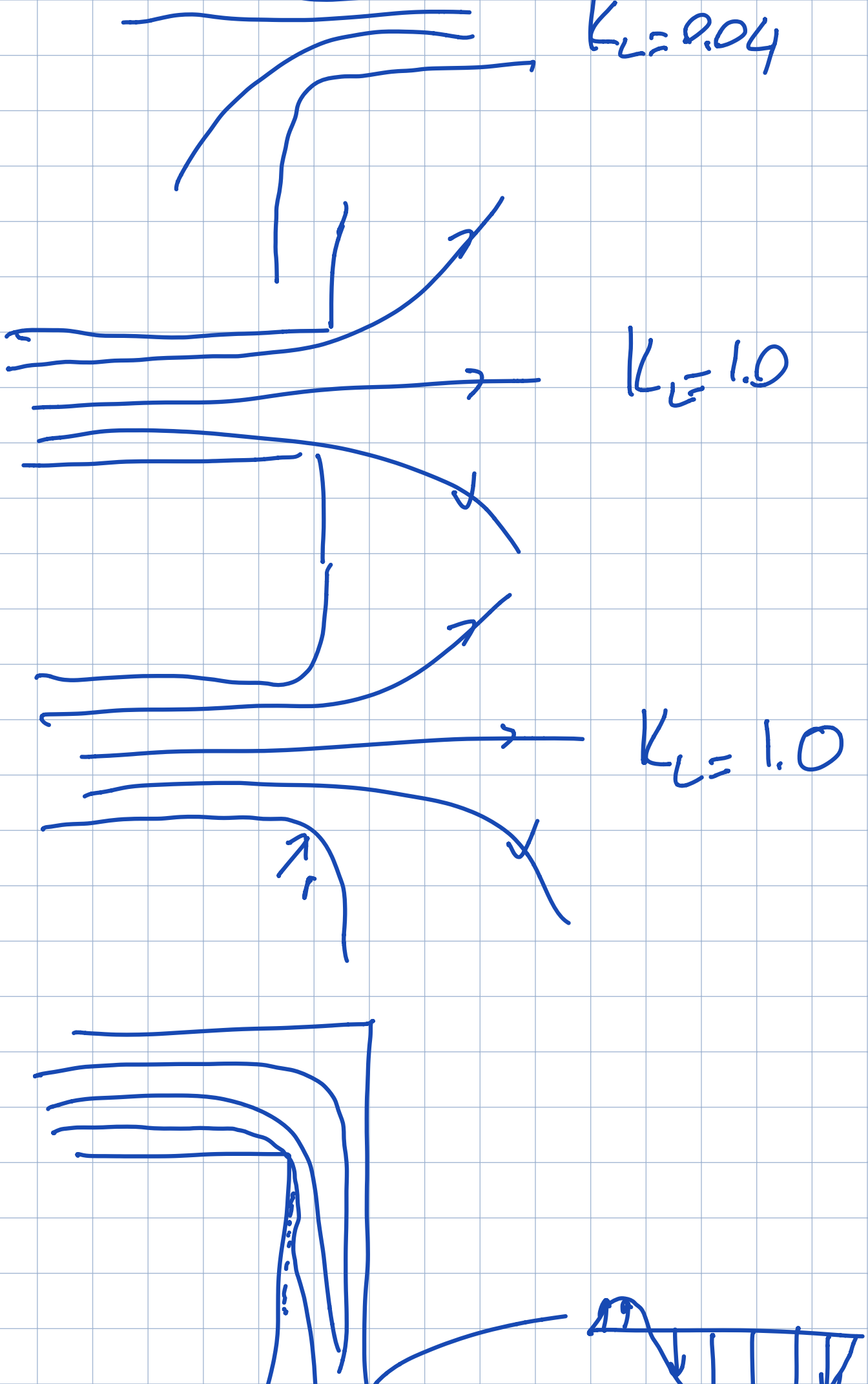
$$h_{L\,major} = f \cdot \frac{l}{D} \frac{V^2}{2g}$$

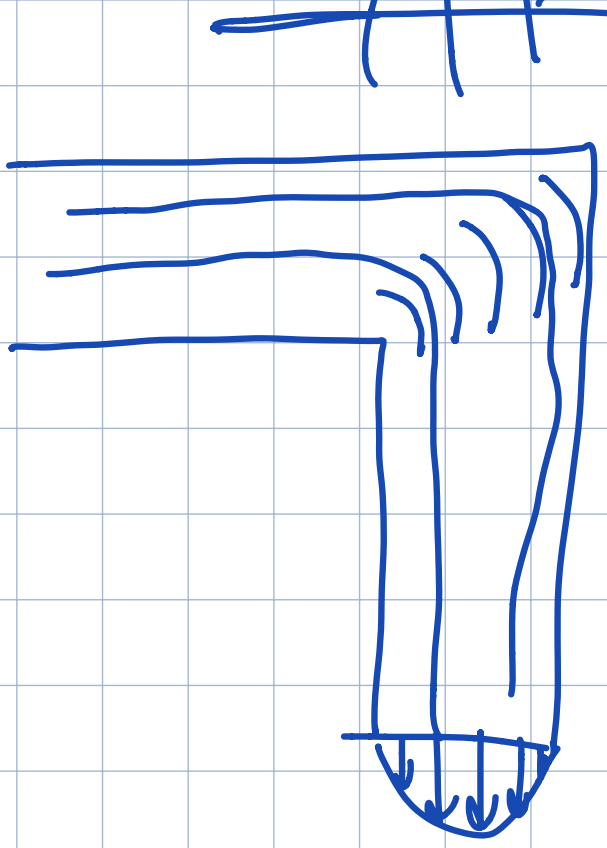
$$K_L = f \cdot \frac{l_{eq}}{D} \Rightarrow l_{eq} = \frac{K_L \cdot D}{f}$$

K_L entrance flow






$K_L = 0.04$
$K_L = 1.0$
$K_L = 1.0$



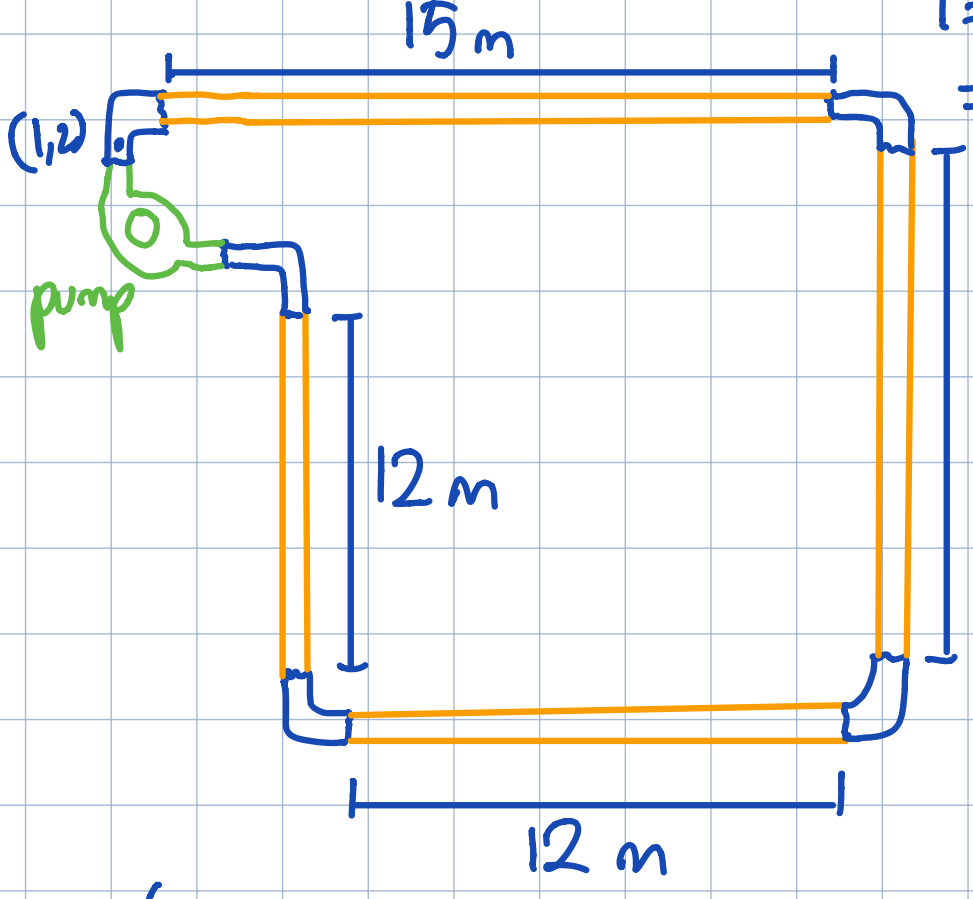
$K_L = 1.0$

$K_L = 0.25$

		K_L
Elbows	90°, threaded	1.5
	flanged	0.3
Valves	Globe, fully open:	10
	Gate valve :	0.15
	Ball valve :	0.05

Example: A pump produces a constant flow rate of 0.4 m^3/s in the closed-loop shown below, which is used in experimental characterization of scaled-down model naval vehicles. The pipes are all 1m in diameter, and made of concrete with a equivalent roughness of 2 mm. Each of the 90° elbows has a loss coefficient of 0.4. Determine the power required?

$l = 15+13+12+12$
$= 52\ m$



$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2 + h_L$$

$h_L = h_{L_{major}} + h_{L_{minor}}$

1=2 $P_1 = P_2$ $V_1 = V_2$ $z_1 = z_2$

$$h_p = h_{L\,major} + h_{L\,minor}$$

$$\underbrace{0.4}_{Q} = V \cdot \underbrace{\frac{\pi}{4}(1)^2}_{A} \Rightarrow \boxed{V = 0.51\ m/s}$$

$$f = f_{xn}\left(Re, \frac{\varepsilon}{D}\right) \qquad Re = \frac{\rho V D}{\mu}$$

$$f = f_{xn}(446{,}000,\ 0.002) \qquad = \frac{(999)(0.51)(1)}{(1.12\times10^{-3})}$$

$$Re \cong 446{,}000$$

$$f = 0.023$$

$$\frac{\varepsilon}{D} = \frac{0.002}{1} = 0.002$$

$$h_p = f \cdot \frac{l}{D}\frac{V^2}{2g} + \sum K_L \cdot \frac{V^2}{2g}$$

$$= (0.023)\frac{(52)}{(1)}\frac{(0.51)}{2(9.81)} + (5)(0.4)\frac{(0.51)}{2(9.81)}$$

$$h_p = 0.016 + 0.0265$$

$$\boxed{h_p = 0.042\ m}$$

$(h_p)\ g \cdot m$

$S\ VA$

$\gamma \quad Q$

$h_p \rightarrow P$

$h_p \cdot g \cdot m$

$h_p \cdot \gamma \cdot Q$

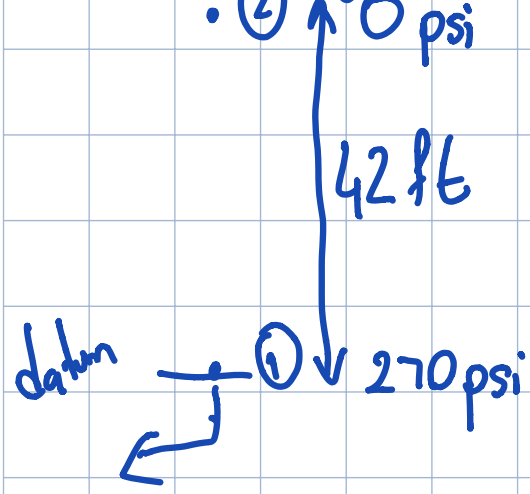
$$P = (0.042)(9.81)(999)(0.4)$$

$$P = 164.6\ W$$

In a 30 story residential building (42 ft high) 1" diameter schedule 40 PVC pipes (max operating pressure of 270 psi) are used. The flow rate of water in the pipes are 0.005 cfs (2.2 gallon/minute). Determine the maximum length of pipes that can be used if you neglect the minor losses?

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{L_{major}}$$

$h_{L_{minor}} = 0$



$z_1 = 0$

$z_2 = 42\text{ ft}$

$V_1 = V_2$

$$(270\text{ psi})(144)\frac{\text{in}^2}{\text{ft}^2}$$

$$\left(\frac{0.033}{D}\right)\left(\frac{L}{D}\right)\frac{V^2}{2g} = 0.92$$

$$\frac{P_1 - P_2}{62.4\ \frac{lb}{ft^3}} = 42 + f \cdot \frac{L}{D} \cdot \frac{V^2}{2g}$$

$D = 1/12$ ft, $g = 32.2$

$$f = f_{xn}\left(Re, \frac{\epsilon}{D}\right) \qquad Re = \frac{\rho V D}{\mu}$$

$$Q = V \cdot A \qquad 0.005\ \frac{ft^3}{s} = V \cdot \frac{\pi}{4}\left(\frac{1}{12}\right)^2$$

$$V = 0.92\ ft/s$$

$$Re = \frac{\left(1.94\ \frac{slug}{ft^3}\right)\left(0.92\ \frac{ft}{s}\right)\left(\frac{1}{12}\right)ft}{2.34 \times 10^{-5}\ \frac{lb\,s}{ft^2}}$$

$$Re \approx 6350 \Longrightarrow f = f_{xn}(6350, 0)$$

$$f = 0.033$$

$$\ell = 111{,}600\ ft \qquad 1\ mile = 5280\ ft$$

$$\underline{\ell \cong 21.1\ miles}$$

1" schedule 40 PVC = 1440

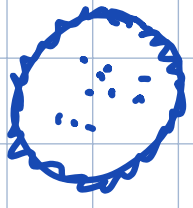
Non-Circular Ducts:

$$Re = \frac{\rho V D_H}{\mu}$$

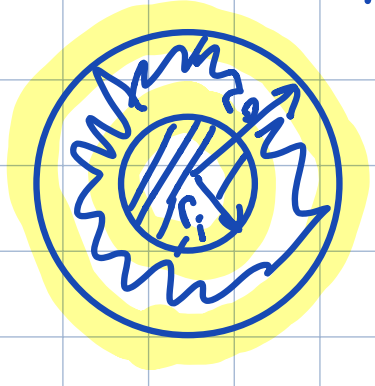
D_H = hydraulic diameter

$$D_H = \frac{4\,(\text{cross-sectional area})}{\text{wetted perimeter}}$$

$$D_H = \frac{4\left(\frac{\pi D^2}{4}\right)}{\pi D} = D$$



Annulus



$$D_H = \frac{4\left(\pi (r_o)^2 - (r_i)^2\right)}{2\pi (r_o) + 2\pi (r_i)}$$

$$D_H = \frac{2\cancel{4}\cancel{\pi}(r_o^2 - r_i^2)}{\cancel{2\pi}(r_o + r_i)}$$

$$D_H = \frac{2(r_o - r_i)\cancel{(r_o + r_i)}}{\cancel{r_o + r_i}}$$

$$D_H = 2(r_o - r_i)$$

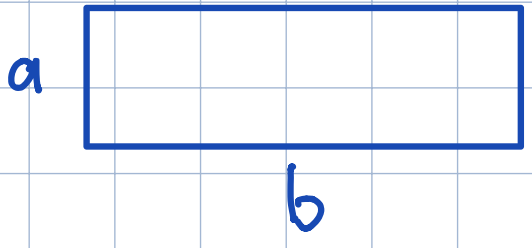
$$\boxed{D_H = D_o - D_i}$$

$$f = \frac{64}{Re}^{C}$$

for annulus

D_i/D_o	C
0.1	89
⋮	⋮
1	96

Rectangle



$$D_H = \frac{4(a \cdot b)}{2a+2b} = \frac{2ab}{a+b}$$

a/b	C
0.1	84
1	56