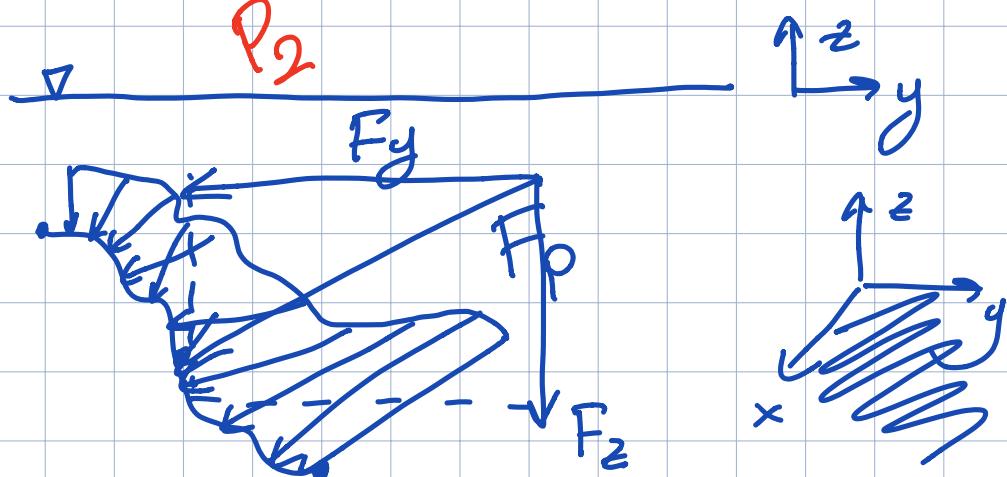


Pressure Forces on submerged surfaces:

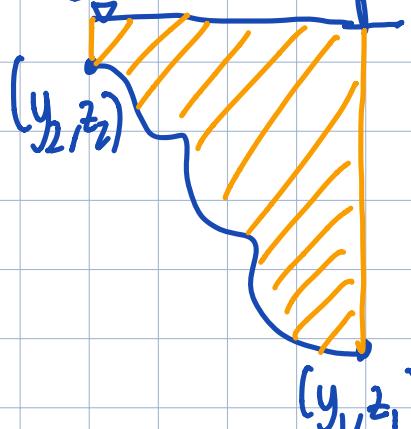


Dimension into the screen is w .

$$\vec{F}_p = F_y (-\hat{j}) + F_z (-\hat{k})$$

$$|F_p| = \sqrt{(F_y)^2 + (F_z)^2}$$

Vertical Component of the pressure force (F_z)

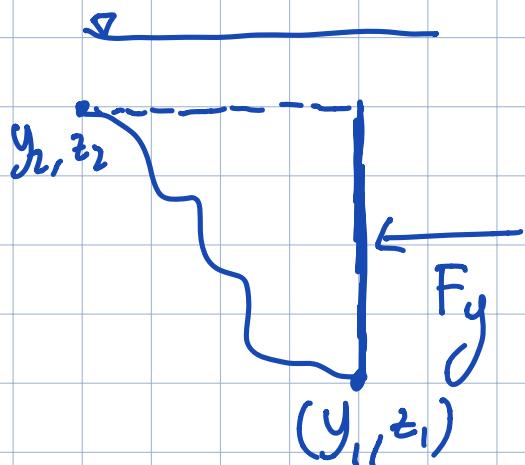


Vertical component of the pressure force is the weight of the fluid above the surface all the way to free surface

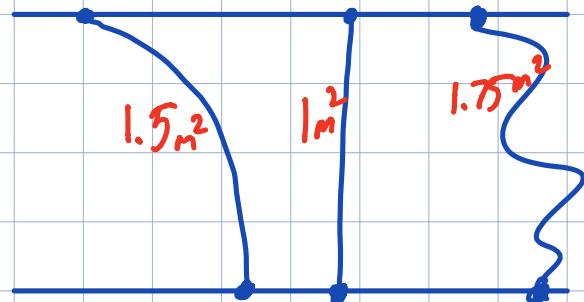
$\Gamma \dots \nabla_{1,1} z_2 r / y_{2,1}$ ↳ real,imaginary

$$+z = W = \rho \cdot V = \iiint \gamma \cdot w \cdot dy dz$$

Horizontal Component of the pressure force (F_y)



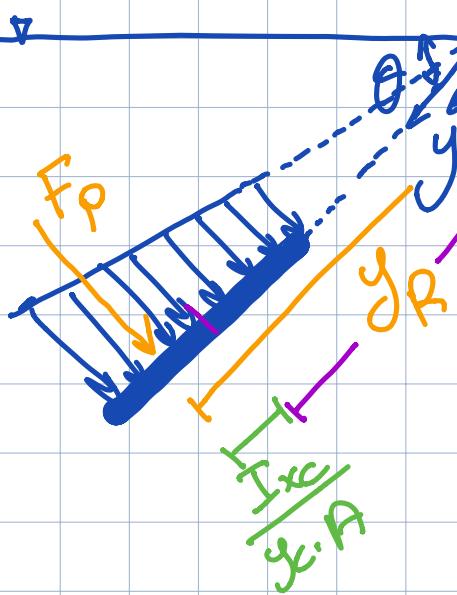
F_y is the force exerted on the vertical projected area in the $-y$ direction by the adjacent fluid.



All these three surfaces will have the same F_y

If the free surface has a pressure of P_2 , the effect will be to increase horizontal and vertical forces by P_2 times vertically and horizontally projected areas, respectively.

Center of Pressure for submerged plates



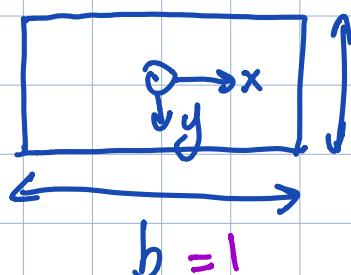
$$y_P = y_C + \frac{I_{xc}}{y_c \cdot A} y > y_C$$

I_{xc} = second moment of area
(moment of inertia)

$$x_P = x_C + \frac{I_{xyc}}{y_c \cdot A}$$

if the area is symmetric w.r.t x or y

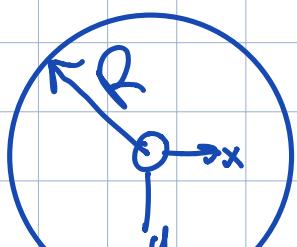
centroidal axes $\Rightarrow I_{xyc} = 0 \Rightarrow \underline{\underline{x_P = x_C}}$



$$A = b \cdot a$$

$$\underline{\underline{I_{xc} = \frac{1}{12} b a^3}}$$

$$I_{xyc} = 0 \Rightarrow \underline{\underline{x_P = x_C}}$$



$$A = \pi R^2$$

$$\underline{\underline{I_{xc} = \frac{\pi R^4}{4}}}$$

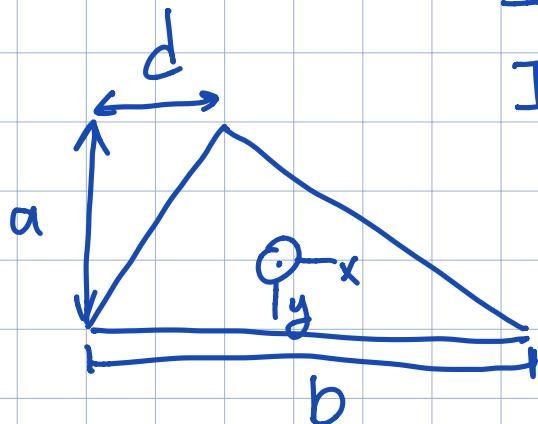
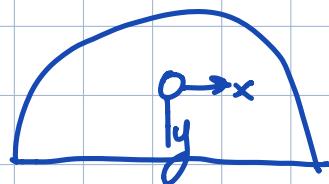
$$\delta$$

$$I_{xc} = \frac{\pi}{4}$$

$$I_{xyc} = 0 \Rightarrow x_R = x_C$$

$$A = \frac{\pi R^2}{2}$$

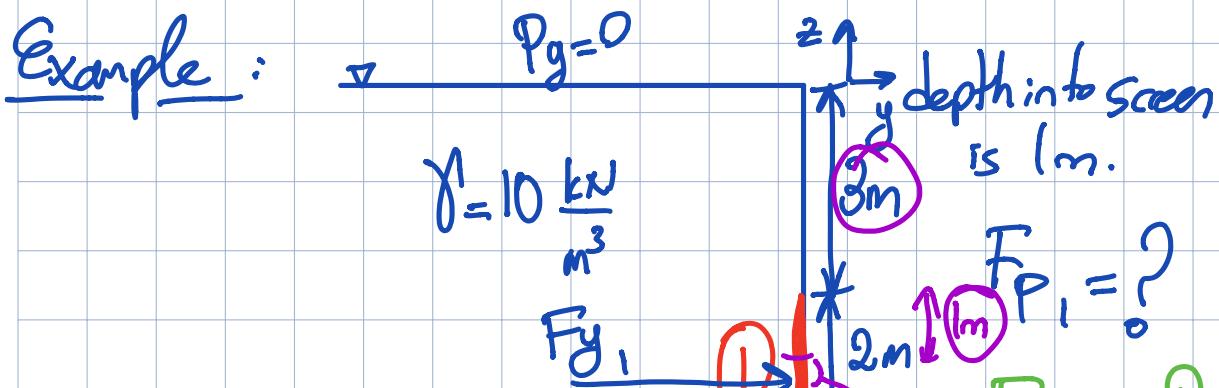
$$I_{xc} = 0.11 R^4$$

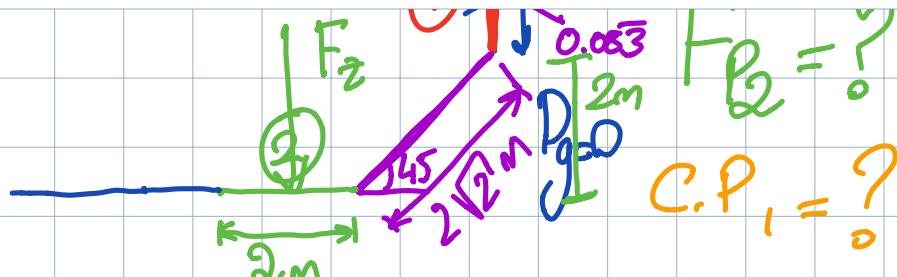


$$A = \frac{a \cdot b}{2}$$

$$I_{xc} = \frac{ba^3}{36}$$

$$I_{xyc} = \frac{ba^2}{72} (b - 2d)$$





$$C.P_1 = ?$$

$$C.P_2 = ?$$

$$F_{P1} = F_y (J)$$



Integration Method:

$$\int dF_y = P_g(z) \cdot dA = \int P_g(z) \cdot dz$$

$$F_y = \int_{\text{area}} P_g(z) dz = \int_{-5}^{-3} -8 \cdot z dz$$

$$= -8 \cdot \frac{z^2}{2} \Big|_{-5}^{-3}$$

$$= -(10,000) \left[\frac{(-3)^2}{2} - \frac{(-5)^2}{2} \right]$$

$$4.5 - 12.5$$

$$= 80,000 \text{ N} = \underline{\underline{80 \text{ kN}}}$$

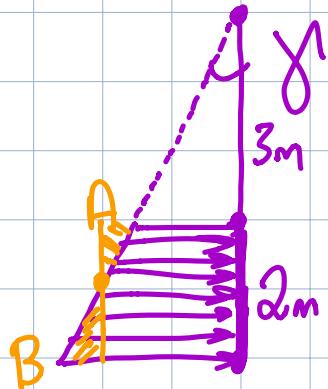
Alternative Method 1: Equation

$$F_y = \gamma (h_c) \cdot A \quad (2-18)$$

↳ geometric center of the surface

$$F_y = (10,000) (4) (2)(1) = \underline{\underline{80,000 \text{ N}}}$$

Alternative Method 2: Geometrical Method
(Pressure Prism Method)



$$F = P_{av} \cdot A$$

$$P_A = \gamma \cdot (3)$$

$$P_B = \gamma \cdot (5)$$

$$P_{av} = \frac{P_A + P_B}{2} = \frac{\gamma(3) + \gamma(5)}{2} = 4(\gamma)$$

$$F_y = 4(\gamma) (2)(1) = \underline{\underline{80,000 \text{ N}}} \\ \hookrightarrow 10,000$$

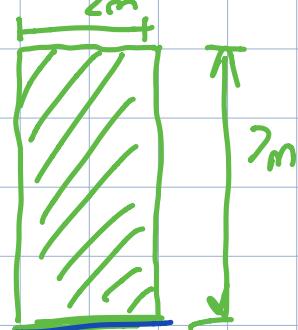
$$\boxed{F_o = 80,000 \text{ N}}$$

$$y_p = y_c + \frac{I_{xx,c}}{y_c \cdot A} \rightarrow \frac{1}{12} \cdot \frac{(1)(2)^3}{8}$$

y_c

$$y_p = 4 + \frac{1}{12} = 4.083 \text{ m}$$

b) $F_p = -F_z (k)$



$$W = \gamma \cdot V$$

$$(10,000) \quad (7)(2)(1) \Rightarrow F_z = 140,000 \text{ N}$$

Alternative method | ¹⁴

$$F = P \cdot A$$

$$\underline{\underline{7}} \underline{\underline{8}} \underline{\underline{(2)(1)}} \Rightarrow$$

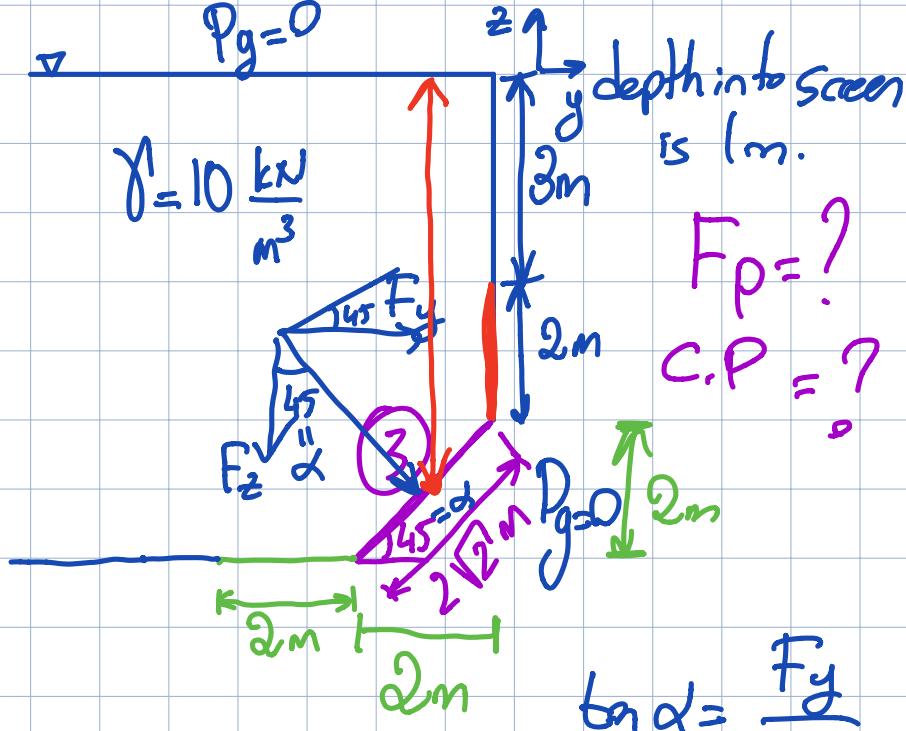
$$P = \gamma \cdot 7 = \underline{\underline{7\gamma}}$$

$$F_z = 140,000 \text{ N}$$

$$\underline{\underline{F_{P_2} = (-140,000 \hat{k})}}$$

$$\underline{\underline{y_c = 7 \text{ m}}}$$

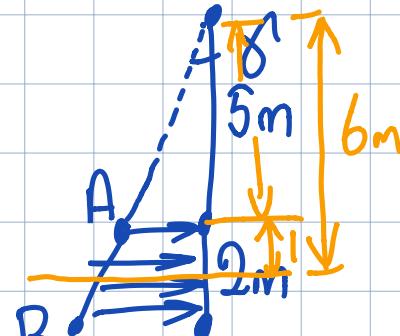
Example : $\nabla \rho g = 0$



$$F_p = F_y (\hat{j}) - F_z (\hat{k})$$

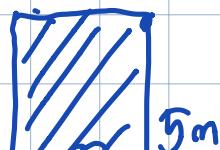
$$\tan \alpha = \frac{F_y}{F_z}$$

$F_y \rightarrow$



$$F_y = (8 \cdot 6) \cdot (2)(1) = \underline{\underline{120,000 \text{ N}}}$$

$10,000 \frac{\text{N}}{\text{m}^3}$



$$|F_z| = w$$



$$\begin{aligned}
 &= \gamma \cdot H \\
 &= (10,000) \cdot H_1 + H_2 \\
 &= (5)(2)(1) \frac{10}{2} + (2)(2)(1) \frac{6}{2} \\
 |F_z| &= W = (10,000)(12) \\
 &= \underline{\underline{120,000 \text{ N}}}
 \end{aligned}$$

Alternative Method 1: use α of the inclined plate to relate F_y and F_z . Find only one component.

Alternative Method 2:

$$\begin{aligned}
 |F_p| &= \gamma |h_{cl}| \cdot A \\
 &\rightarrow \text{geometric center of the surface (z direction)}
 \end{aligned}$$

$$|F_p| = (10,000) (6) (2\sqrt{2}) (1)$$

$$= \underline{\underline{169,705 \text{ N}}} \Rightarrow 169.7 \text{ kN}$$

$$\Sigma (120\text{m}) \uparrow (120\text{m}) \downarrow$$

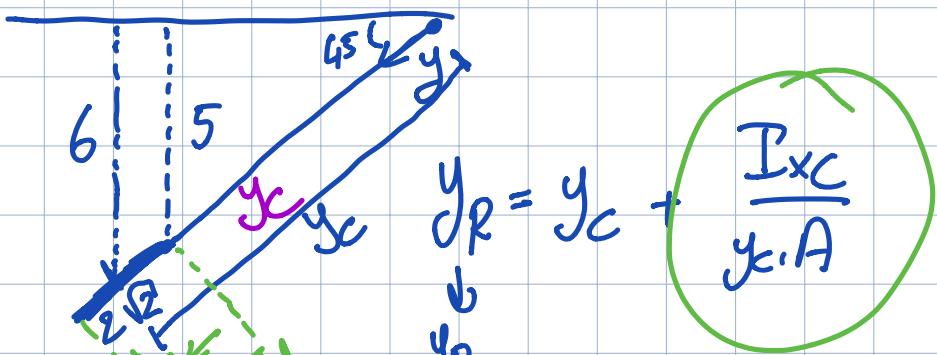
$$P = (120,000) \vee (120,000)$$

$$|F_p| = \sqrt{(120,000)^2 + (120,000)^2}$$

$$= \underline{169,705 \text{ N}}$$

$$\theta = 45^\circ$$

C.P calculation



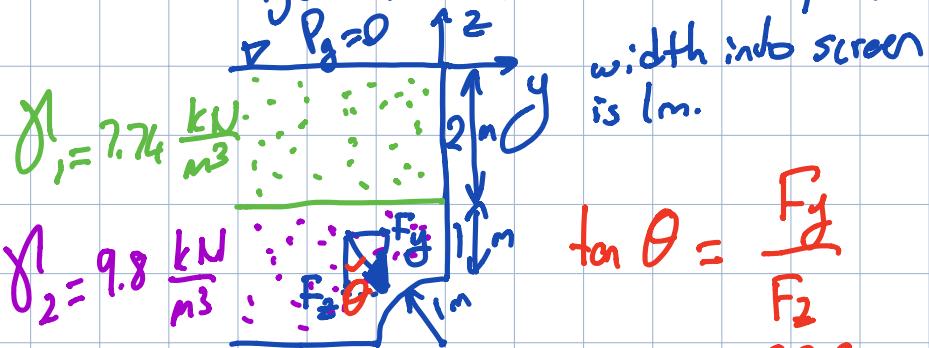
$$\sin 45^\circ = \frac{6}{y_C} \Rightarrow \frac{\sqrt{2}}{2} = \frac{6}{y_C}$$

$$y_C = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2} \cdot \sqrt{2}}{2} = 6\sqrt{2}$$

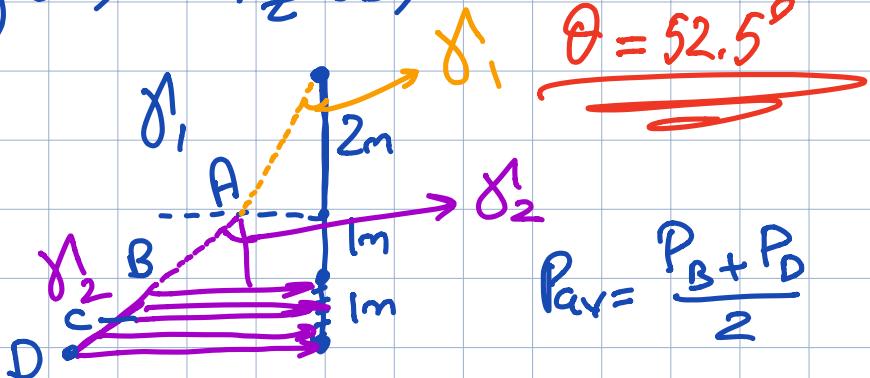
$$\frac{I_{x_C}}{y_C \cdot A} = \frac{\frac{1}{12} (1)(2\sqrt{2})^3}{(6\sqrt{2})(2\sqrt{2})(1)} = \frac{1}{12} \cdot \frac{8 \cdot 2 \cdot \sqrt{2}}{24}$$

$$= \frac{\sqrt{2}}{18} = \underline{0.08 \text{ m}}$$

Example: Find the hydrostatic force on the quarter-circle.



$$F_p = F_y(\hat{j}) - F_z(\hat{k})$$



$$P_A = \gamma_1(2) = 15,480 \text{ Pa}$$

$\hookrightarrow 7740 \frac{\text{N}}{\text{m}^3}$

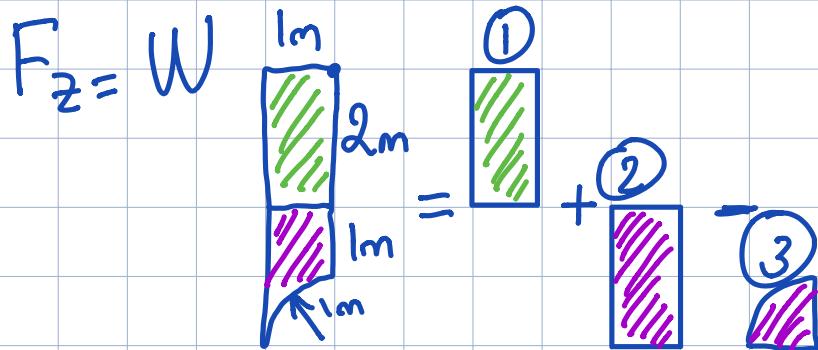
$$P_C = (15,480) + (9800)(1.5)$$

$$P_C = 30,180 \text{ Pa}$$

$$F_y = P_c \cdot A \xrightarrow{(VCI)} \Rightarrow F_y = 30,180 \text{ N}$$

$\hookrightarrow (30,180)$

$$\underline{\underline{F_y = 30.2 \text{ kN}}}$$



$$F_z = \gamma_1 \cdot V_1 + \gamma_2 V_2 - \gamma_2 V_3$$

$$(7740) \frac{(1)(2)(1)}{(9800)} (9800) \frac{(1)(2)(1)}{\frac{\pi(1)^2}{4} \cdot (1)}$$

$$= (7740)(2) + 9800 \left(2 - \frac{\pi}{4} \right)$$

$$= 23,175 \text{ N} \Rightarrow \underline{\underline{F_z = 23.2 \text{ kN}}}$$

$$F_p = 30.2 \text{ kN}(\uparrow) - 23.2 \text{ kN}(\downarrow)$$

—

+ ② - ③

$$\overline{F_H = 30,180 \text{ N}}$$

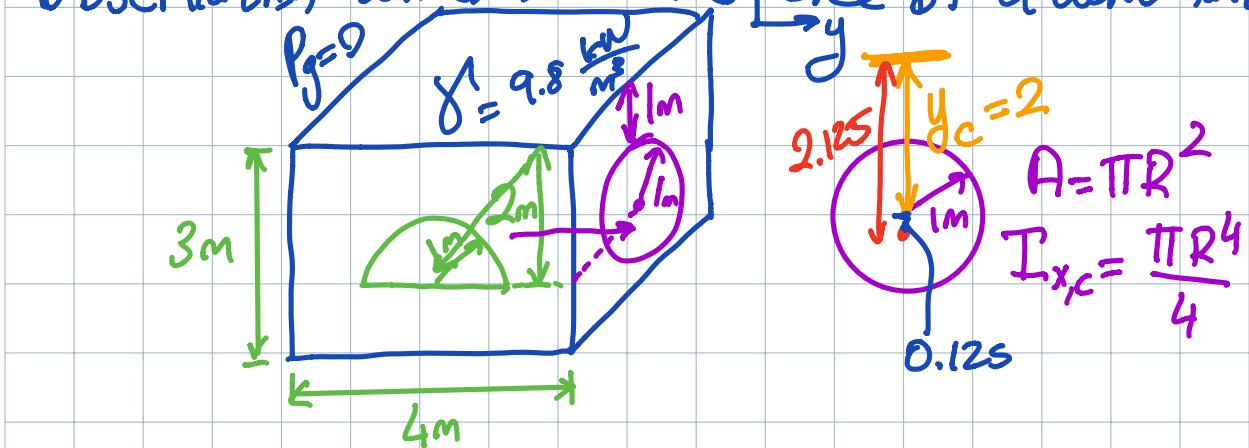
$$(7740)(1)(2) + (9800)(2) - (9800) \left(\frac{\pi(1)^2}{4} \right)$$

$$(7740)(2) + 9800(2 + \frac{\pi}{4})$$

7699

$$\boxed{F_v 23,179 \text{ N}}$$

Example: Find the hydrostatic forces and centers of pressures for the semi-circular and circular observation windows on the side of a water tank.



$$F_{Pc} = F_y (\hat{j})$$

$$F_y = \gamma \cdot h_c \cdot A$$

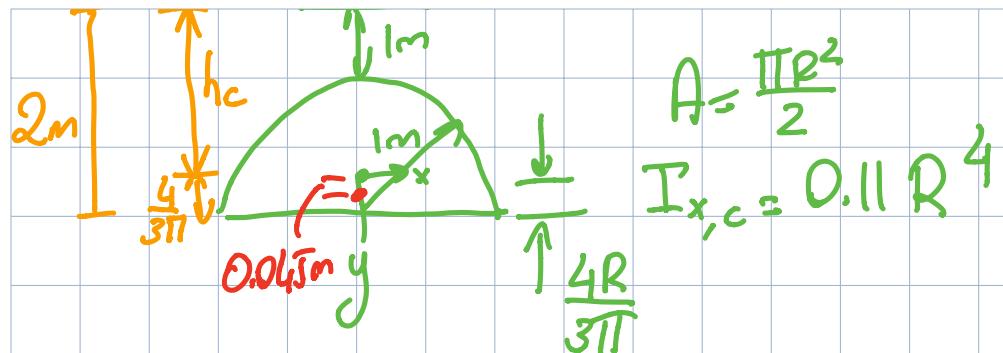
$$9800 \quad 2 \quad \pi (1)^2$$

$$\underline{F_y = 61,575 \text{ N} = 61.6 \text{ kN}}$$

$$F_{Pc} = 61.6 \text{ kN} \hat{j}$$

$$y_R = y_c + \frac{I_{x,c}}{\frac{y_c \cdot A}{2}} \rightarrow \frac{\pi}{4} (1)^4$$

$$= 2 + \frac{1}{\frac{0.125}{8}} = \underline{2.125 \text{ m}}$$



$$F_{P_S} = F_x \uparrow$$

$$F_x = \gamma \cdot h_c \cdot A$$

$$= F_x = 24,245 \text{ N}$$

$$= F_x = 24.2 \text{ kN}$$

\downarrow \downarrow \downarrow

$$9800 \quad \left(2 \cdot \frac{4}{3\pi}\right) \quad \frac{\pi(1)^2}{2}$$

$$\boxed{F_{P_S} = 24.2 \text{ kN} \uparrow}$$

$$y_R = y_c + \frac{I_{x,c}}{y_c \cdot A} \rightarrow 0.11(1)^4$$

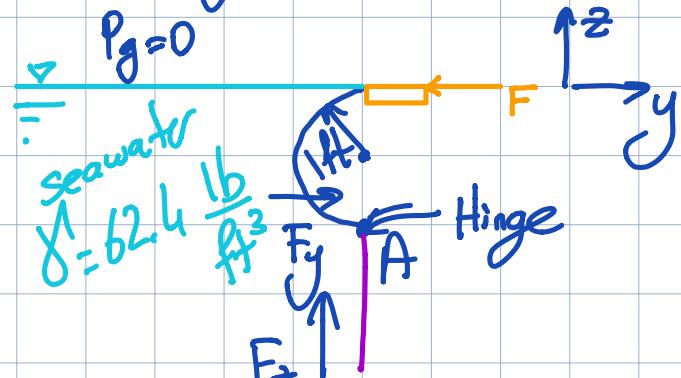
$$= 1.575 \quad \downarrow \quad \downarrow$$

$$1.575 \quad 1.575$$

$$\underline{\underline{y_R}} = 1.575 + 0.045 = \underline{\underline{1.62 \text{ m}}}$$

Example: Find the Force (P) required to hold the semi-circular gate hinged at point A in place?

depth into the screen is 1 ft



$$F_p = F_y (\hat{j}) + F_z (\hat{k})$$

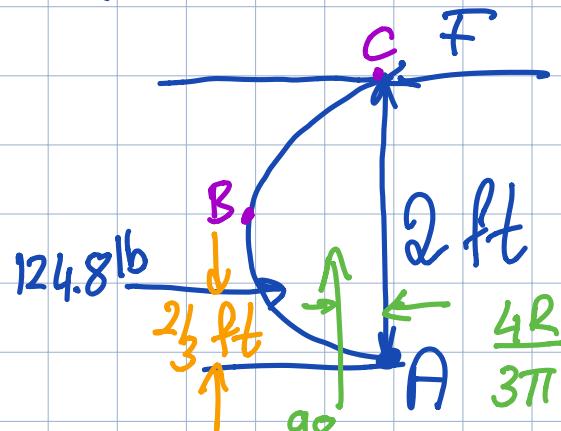
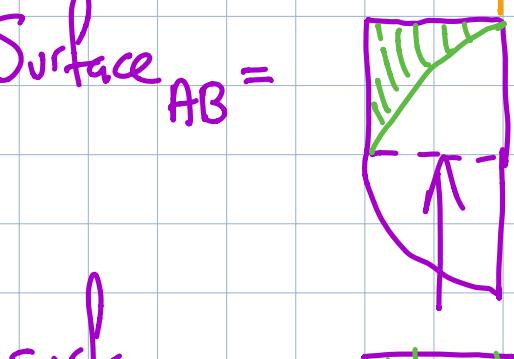
$$F_p = (124.8) \hat{j} + (98) \hat{k}$$

$$F_y \Rightarrow F_y = \gamma \cdot h_c \cdot A = 124.8 \text{ lb}$$

$(62.4) \quad (1) \quad (2)(1)$



$$\text{Surface}_{AB} =$$



$$F_z = \gamma \cdot H$$

$$62.4 \frac{\text{lb}}{\text{ft}^3} \left(\frac{\pi(1)^2}{2} \right) (1)$$

Surf face

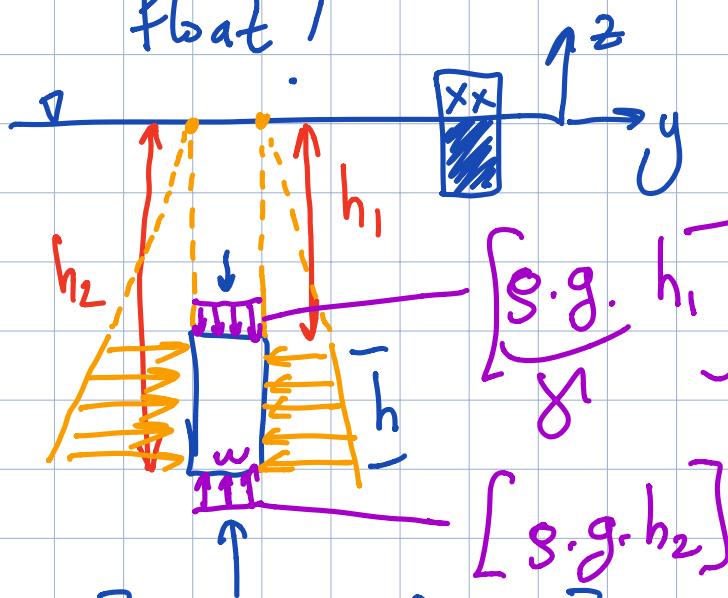
$$BC = \frac{1}{2} \cdot \frac{\pi}{2} \cdot 1^2 = \frac{62.4 \pi}{2} = 98 \text{ lb}$$

$$(+M_A) (124.8) \left(\frac{2}{3}\right) + (98) \left(\frac{4}{3\pi}\right) - F.(2) = 0$$

$$F = 62.4 \text{ lb}$$

Buoyancy and Stability:

Why fully-submerged or partially-submerged bodies float?



$$[\gamma \cdot g.c. h_1] w.d$$

$$[\gamma \cdot g.c. h_2] \cdot w.d$$

$$[\gamma \cdot h_2] w.d - [\gamma \cdot h_1] w.d$$

$$\gamma w.d [h_2 - h_1] = \frac{\gamma \cdot w.d \cdot h}{1}$$

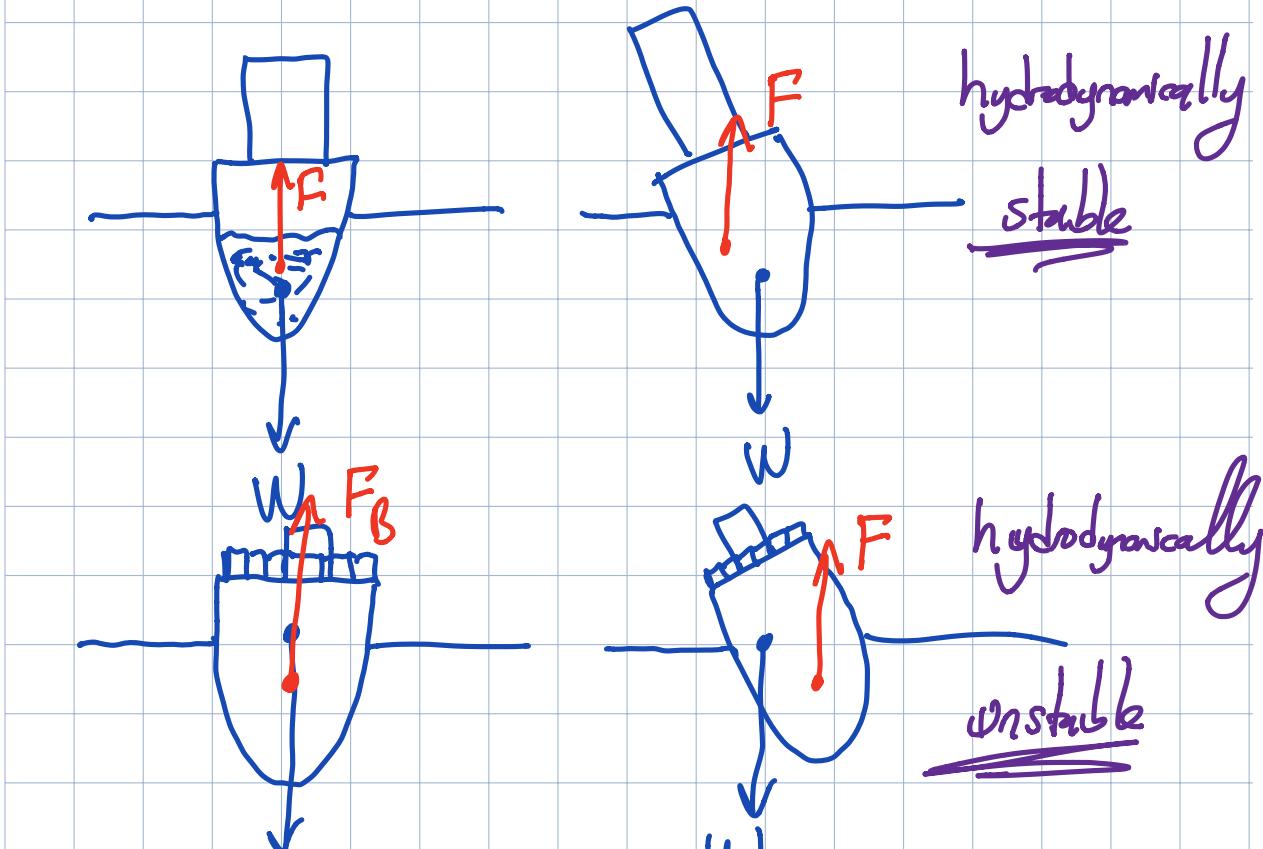
$$\underline{F_B = \gamma \cdot V}$$

V

Archimedes' Principle: F_B acting on a solid submerged or partially submerged in a fluid is equal to weight of the displaced fluid

→ Buoyant force passes through the centroid of

the displaced volume



W

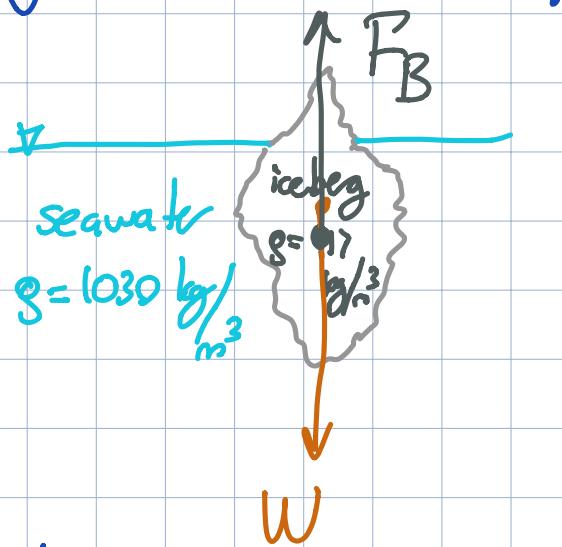
W

Example: What percent of the iceberg volume

is under water?

$$\rho_{\text{iceberg}} = 917 \text{ kg/m}^3; \rho_{\text{seawater}} = 1030 \frac{\text{kg}}{\text{m}^3}$$

$$F_B = W$$
$$W = \gamma_{\text{iceberg}} + \gamma_{\text{total}}$$
$$F_B = \gamma_{\text{seawater}} + \gamma_{\text{underwater}}$$

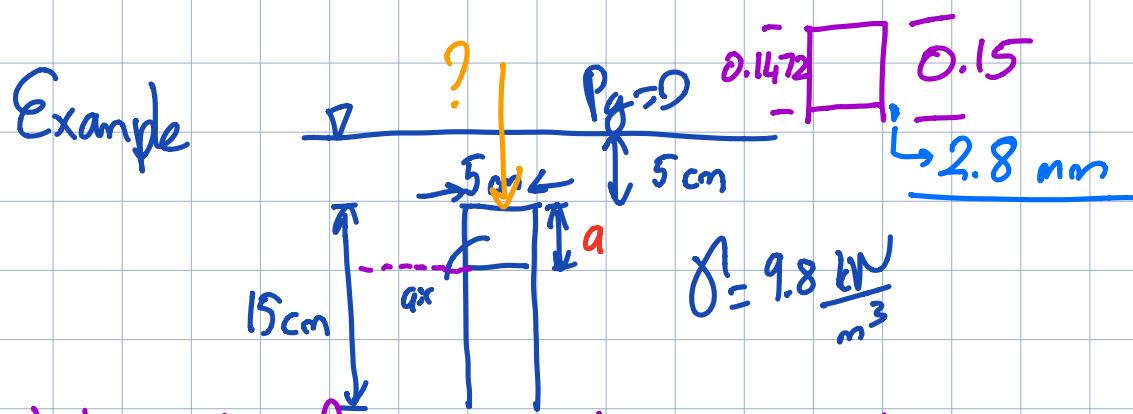


$$\gamma_{\text{iceberg}} \cdot V_{\text{total}} = \gamma_{\text{seawater}} \cdot V_{\text{underwater}}$$

$$\frac{V_{\text{underwater}}}{V_{\text{total}}} = \frac{\gamma_{\text{iceberg}}}{\gamma_{\text{seawater}}} = \frac{\rho_{\text{iceberg}} \cdot g}{\rho_{\text{seawater}} \cdot g}$$

$$\frac{V_{\text{underwater}}}{V_{\text{total}}} = \frac{\text{Iceberg}}{\text{Seawater}} = \frac{917}{1030} = 0.89$$

89% of iceberg volume is under water



What is the force required to keep the 0.1 kg glass submerged at 5 cm below the surface?



$$F + W = F_B$$

$$F = F_B - W \rightarrow m \cdot g$$

$$F = X \cdot \pi (0.025)^2 \cdot 0.098$$

compression isothermal

$$PVT = M.R.T$$

~~$$P_{atm} \cdot (T_f) (0.15) = P_{air} (T_f^2) (a)$$~~

$$a = \frac{P_{atm} (0.15)}{P_a}$$

$$P_a = P_{atm} + \gamma_w (0.05 + a)$$

$$a = \frac{P_{atm} (0.15)}{P_{atm} + \gamma_w (0.05 + a)}$$

$$P_{atm} a + \gamma_w (0.05 a + a^2) = P_{atm} (0.15)$$

$$101325 a + 9800 (0.05 a) + 9800 a^2 = 101325 (0.15)$$

$$9800 a^2 + (101325 + 9800 (0.05)) a - 101325 (0.15) = 0$$

$$9800 a^2 + (101815) a - 15199 = 0$$

$$\text{Matlab roots}([9800, 101815, -15199])$$

$$\underline{a = 0.1472}$$

$$\Sigma \pi (r_{min})^2 (n_{1172} / a^{2.0}) \approx 1$$

$$l = \underbrace{11(0.025)(0.14)}_{\delta} + \underbrace{(0.025) - 0.101}_{\delta}$$

$$F = 1.85 \text{ N}$$

$$T = T_B + U \rightarrow (0.1)(9.81)$$

$$\frac{\pi(0.025)^2 a(9800)}{8! \cdot 0.1472} \text{ kg m/s}^2$$

$$PT = mRT$$

compression isothermal

$$P_{atm} \cdot \cancel{\left(\frac{1}{T_f}\right)} \cdot H = P_a \cancel{\left(\frac{1}{T_f}\right)} a$$

$$a = \frac{P_{atm} \cdot H}{P_a}$$

$$P_a = P_{atm} + \gamma_w \cdot (0.05 + a)$$

$$a = \frac{P_{atm} \cdot H}{P_{atm} + \gamma_w (0.05 + a)}$$

$$a = \frac{(101325)(0.15)}{(101325) + (9800)(0.05 + a)}$$

$$a = 0.147$$

$$101325a + 9800(0.05a + a^2) = (101325)(0.15)$$

$$9800a^2 + (101325 + 9800(0.05))a - (101325) = 0$$

$$9800a^2 + 101815a - 15,199 = 0$$

M Hab roots([9800, 101815, -15,199])