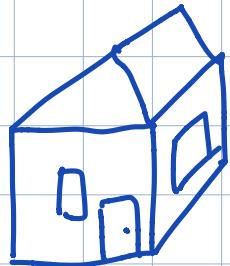


Module 4: Fluid Kinematics:

In fluid statics

$$\sum F = m \cdot a = 0 \\ \rightarrow 0$$



[mass
momentum
energy]

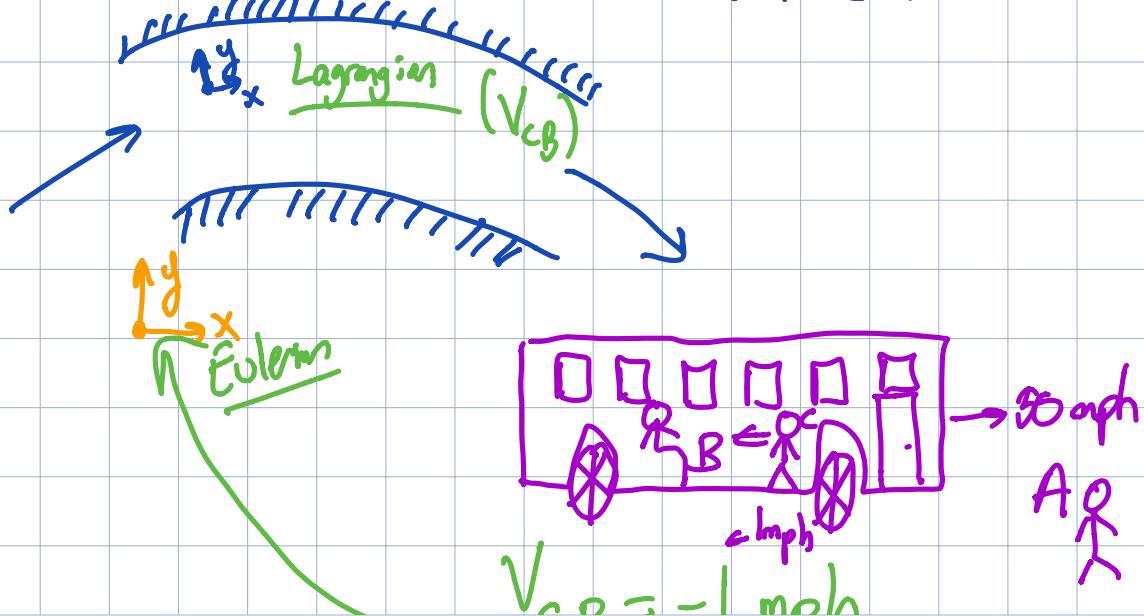
① Reference Frames :

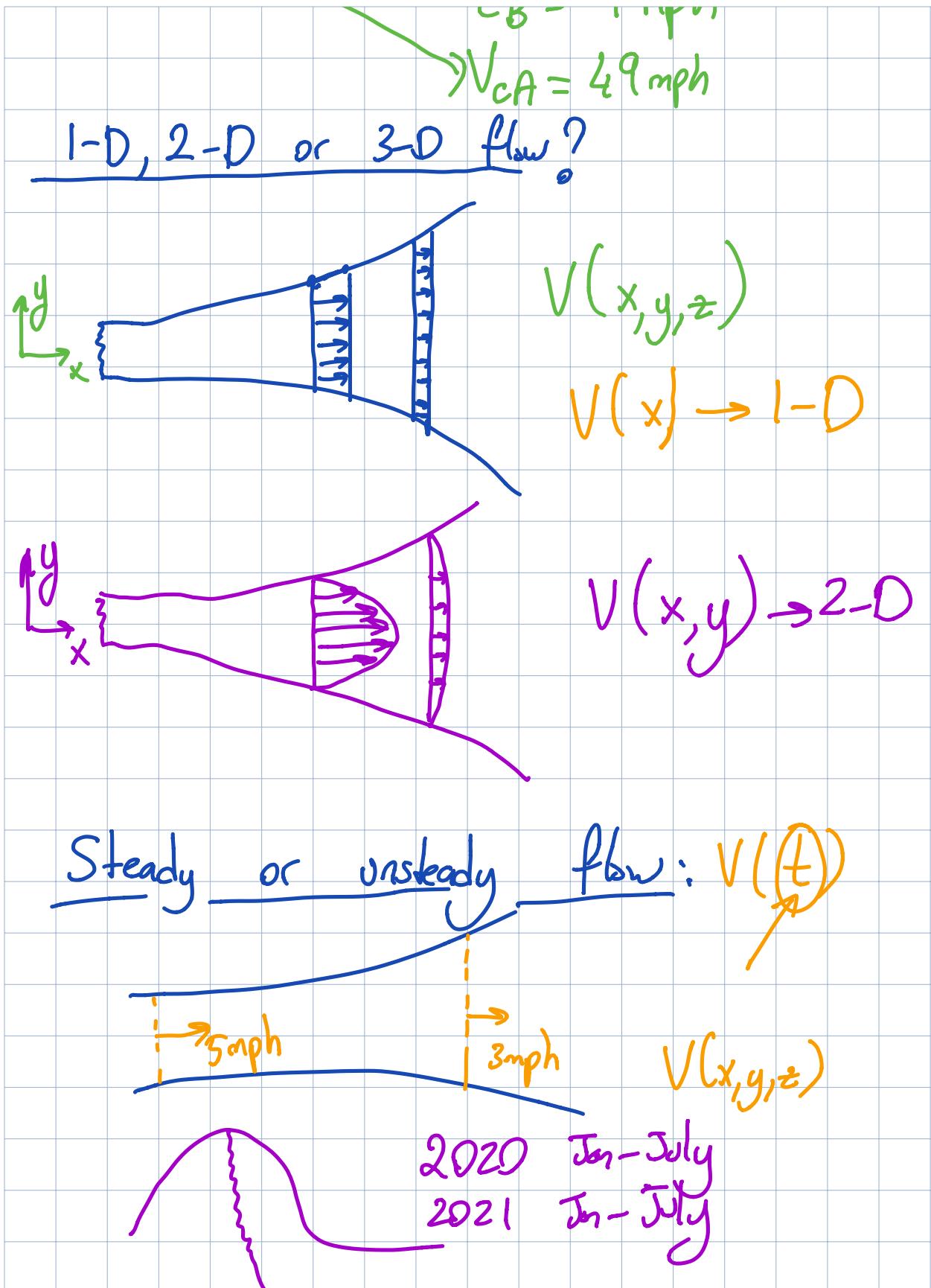
1) Lagrangian Ref. frame : "fixed on a particle in my flow"

Material Derivative

2) Eulerian Ref. frame : "fixed at a certain origin outside the flow"

Derivative

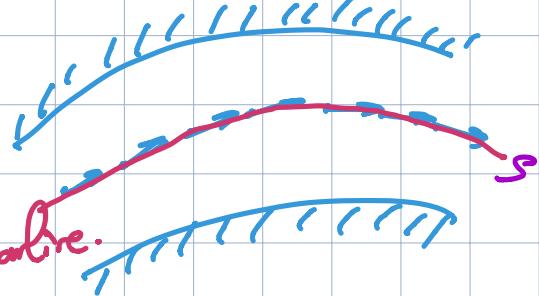




Streamline, Pathline and Streaklines:

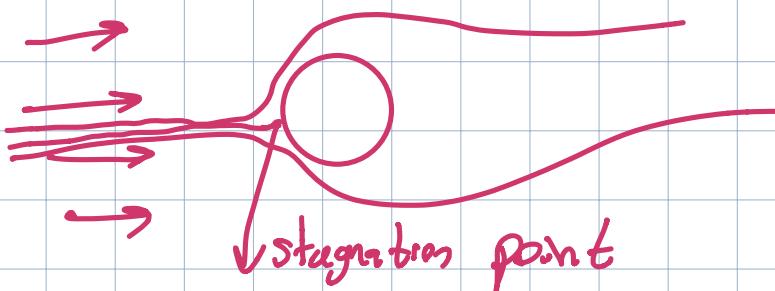
Steady \rightarrow streamline = pathline = streaklines.

unsteady \rightarrow streamline :



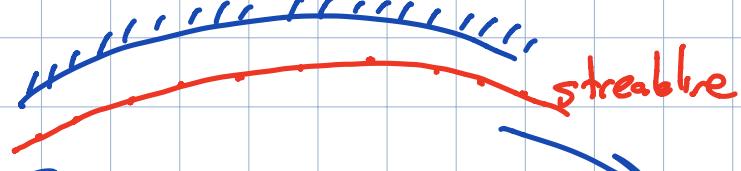
Velocity is tangent to streamlines.

No flow can flow across a streamline



$V=0$ stagnation point

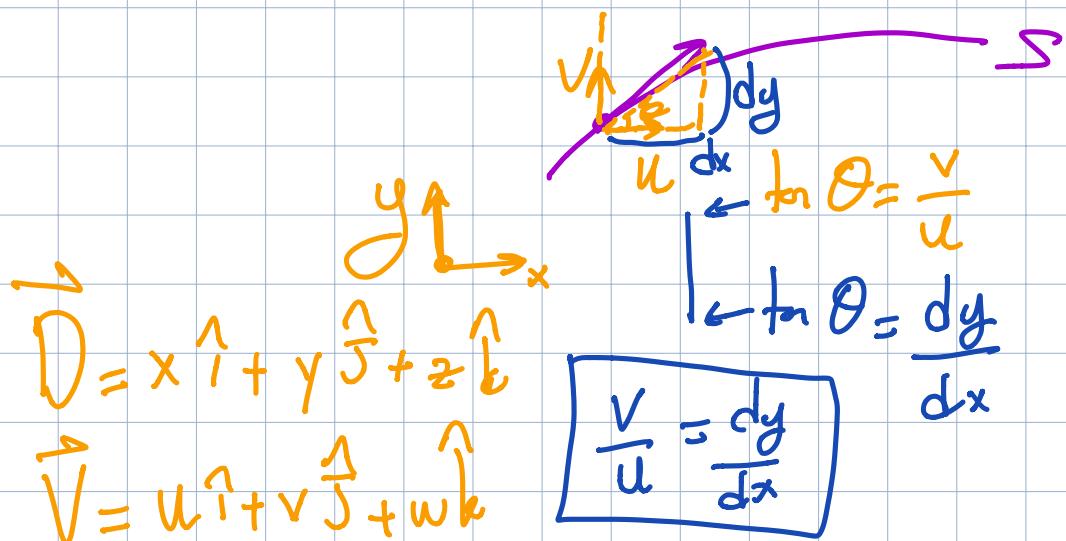
Streakline : Succession of marked particles that originated from a particular point in the flow.



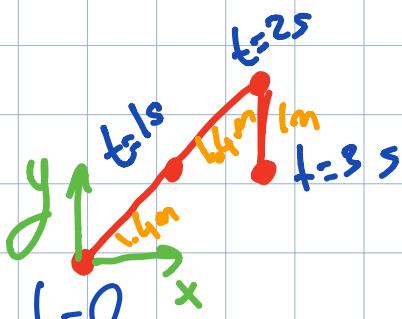


Pathline: Path of the particle travel over time

Equation of streamline



Example: A large drop of a food coloring dye is released into a river flow at $t=0$ sec. The pathline is as shown below. Draw the streamline and the streamline for $t=1$ and 3 seconds



- streamline
- streakline
- pathline

$$\underline{t=1\text{ s}}$$



$$t = 3\text{ s}$$



Acceleration and Material Derivative:

$\vec{a}(\vec{D}, t)$ = time rate of change of $\vec{V}(\vec{D}, t)$

$\vec{V}(\vec{D}, t)$ = time rate of change of $\vec{D}(t)$

$$\vec{D}(t) = x^{\hat{i}} + y^{\hat{j}} + z^{\hat{k}}$$

$$\vec{V}(\vec{D}, t) = \frac{d}{dt}(\vec{D}(t)) = \frac{dx}{dt}^{\hat{i}} + \frac{dy}{dt}^{\hat{j}} + \frac{dz}{dt}^{\hat{k}}$$

$u \rightarrow$ Velocity vector component in x direction

$v \rightarrow$ " " " " " y "

$w \rightarrow$ " " " " " z direction

$$\vec{V}(\vec{D}, t) = u^{\hat{i}} + v^{\hat{j}} + w^{\hat{k}}$$

$$\vec{a}(\vec{r}, t) = \frac{d}{dt} [\vec{v}(\vec{r}, t)]$$

$$= \underbrace{\frac{\partial \vec{v}}{\partial t}}_{\text{local}} + u \underbrace{\frac{\partial \vec{v}}{\partial x}}_{\text{convective}} + v \underbrace{\frac{\partial \vec{v}}{\partial y}}_{\text{acceleration}} + w \underbrace{\frac{\partial \vec{v}}{\partial z}}$$

$$\vec{a} = \underbrace{\frac{\partial \vec{v}}{\partial t}}_{\text{local}} + u \underbrace{\frac{\partial \vec{v}}{\partial x}}_{\text{convective}} + v \underbrace{\frac{\partial \vec{v}}{\partial y}}_{\text{acceleration}} + w \underbrace{\frac{\partial \vec{v}}{\partial z}}$$

$$\vec{a} = \frac{D\vec{v}}{Dt} \quad \left(\frac{D(\dots)}{Dt} = \frac{\partial(\dots)}{\partial t} + u \frac{\partial(\dots)}{\partial x} + v \frac{\partial(\dots)}{\partial y} + w \frac{\partial(\dots)}{\partial z} \right)$$

material derivative

$$\frac{D(\dots)}{Dt} = \frac{\partial(\dots)}{\partial t} + (\vec{v} \cdot \nabla)(\dots)$$

gradient operator

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

Control Volume and Control Surface:

System: An arbitrary volume with fluid particles in it

↳ Control mass \rightarrow no mass crosses the boundary

↳ Control volume (C.V.) \rightarrow mass/momentum/energy cross the boundary

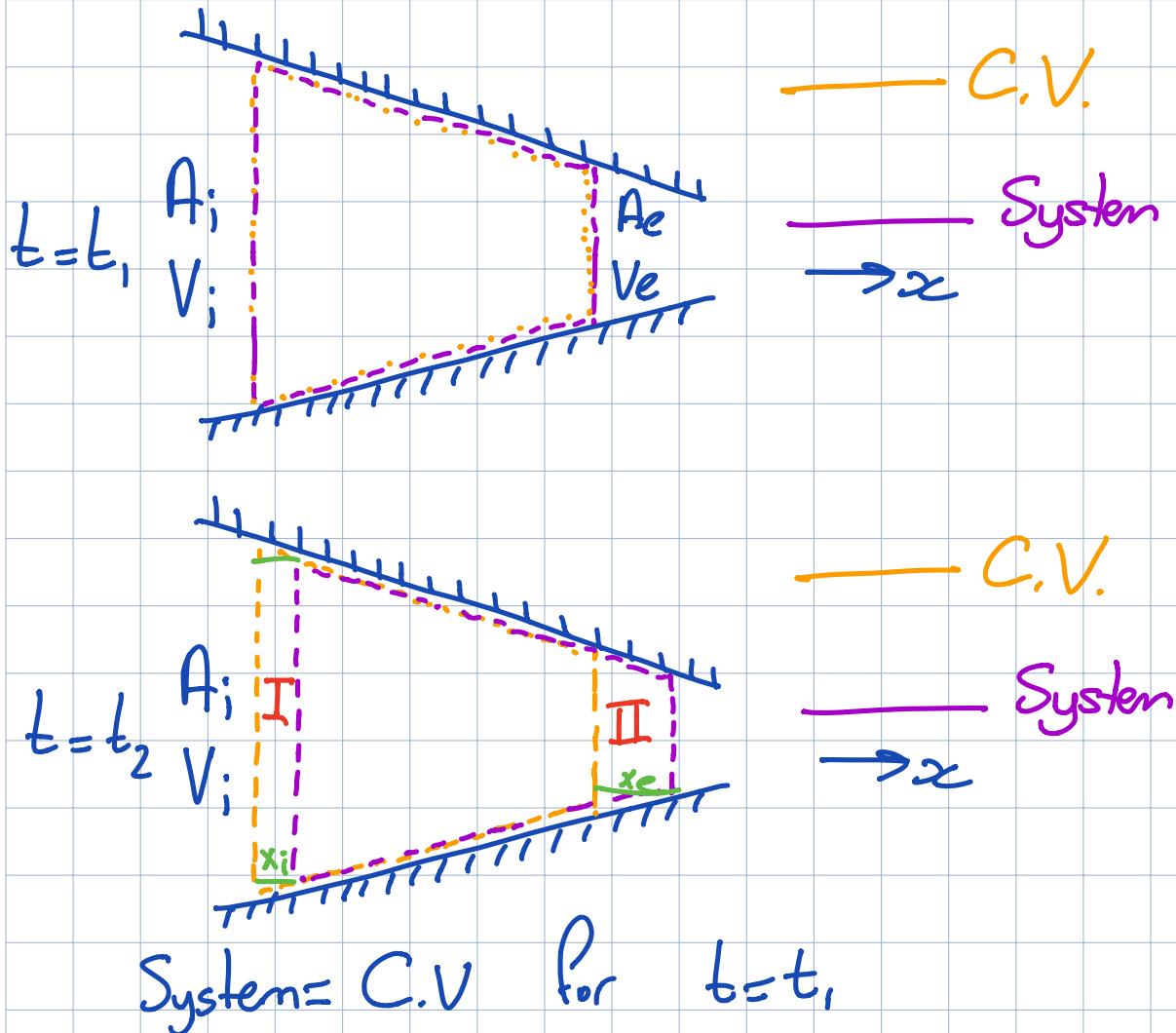
Deforming C.V. \rightarrow shape

Moving C.V. \rightarrow position

control surface
(C.S.)

Reynolds Transport Theorem: (RTT)

Converts properties from Lagrangian to Eulerian
ref. frame.



extensive property = B

$$B_{\text{sys}}(t_1) = B_{\text{CV}}(t_1)$$

$$t_2 = t_1 + \Delta t$$

Fluid molecules left region I, entered region II

$$B_{sys}(t_2) = \underbrace{B_{cv}(t_2)}_{B_{sys}(t_1)} + B_{II}(t_2) - B_I(t_2)$$

$$\frac{\Delta B_{sys}}{\Delta t} = \frac{B_{sys}(t_2) - B_{sys}(t_1)}{\Delta t}$$

$B_{cv}(t_1)$
↓

$$= \frac{B_{cv}(t_2) - B_I(t_2) + B_{II}(t_2) - B_{sys}(t_1)}{\Delta t}$$

$$= \frac{B_{cv}(t_2) - B_{cv}(t_1)}{\Delta t} - \frac{B_I(t_2)}{\Delta t} + \frac{B_{II}(t_2)}{\Delta t}$$

$$\Delta t \rightarrow 0$$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta B_{sys}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{B_{cv}(t_2) - B_{cv}(t_1)}{\Delta t}$$

$\Gamma B_{II}(t_2) \quad B_I(t_2)$

+ $\lim_{\Delta t \rightarrow 0} \left[\frac{\overline{B_I(t_2)}}{\Delta t} - \frac{\overline{B_I(t_1)}}{\Delta t} \right]$

By definition
material derivative $\left(\frac{DB_{sys}}{Dt} \right)$

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \lim_{\Delta t \rightarrow 0} \left[\frac{B_{II}(t_2) - B_{I}(t_2)}{\Delta t} - \frac{B_{II}(t_1) - B_{I}(t_1)}{\Delta t} \right]$$

$$b = \frac{B}{m}$$

$B \rightarrow$ extensive property

$b \rightarrow$ intensive property

$$\lim_{\Delta t \rightarrow 0} B_I(t_2) = b_i \cdot m_i$$

$$S = \frac{m}{V}$$

$$m_i = S_i \cdot V_i$$

$$A_i \cdot x_i$$

$$V_i / \Delta t$$

$$m_i = S_i \cdot A_i \cdot V_i / \Delta t$$

$$\lim_{\Delta t \rightarrow 0} \frac{B_I(t_2)}{\Delta t} = \frac{b_i \cdot S_i \cdot A_i \cdot V_i / \Delta t}{\Delta t} = \dot{B}_{in}$$

$$0 \quad B_{II}(t_1), \dots, \dots, \dot{B}_{in}, \dots, i$$

$$\lim_{\Delta t \rightarrow 0} \frac{\vec{B}_{sys}^{(t+\Delta t)} - \vec{B}_{sys}^{(t)}}{\Delta t} = b e \vec{S}_e \vec{H}_e \vec{V}_e = \vec{B}_e$$

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{CV}}{\partial t} + (\dot{B}_e - \dot{B}_n)$$

Lagrangian
ref. frame

Eulerian ref. frame

General form RTT

Integral form of RTT

$$\frac{DB}{Dt} = \frac{\partial}{\partial t} \iiint_{C.V.} b \cdot g \cdot dV + \iint_{C.S.} b \cdot g (\vec{V} \cdot \hat{n}) dA$$

Lagrangian

\vec{B} ~~exist~~ mass, momentum, energy $b = \frac{B}{m}$

$$\frac{DB}{Dt} = \frac{\partial}{\partial t} \iiint_{C.V.} b \cdot g dV + \iint_{C.S.} b \cdot g (\vec{W} \cdot \hat{n}) dA$$

$$\vec{W} = \vec{V} \text{ for stationary C.V.}$$

+V exit
-V inlet
Q otherwise

\vec{W} = velocity measured w.r.t C.S

