

Conservation of Energy:

Consider a thermodynamic system of a fluid
Total energy of the system

$$E = E_k + E_p + U$$

^ ^ |
kinetic potential Internal
energy energy (molecular)
 |
 n n .

Classification of Energy transfer to and from the system

1) Heat Transfer (Q) from the surrounding to the system
+ Q implies $E \uparrow$

2) Work (W_T) from the system to the surrounding
+ W implies $E \downarrow$

So, for a system that goes from state 1

to 2:

$$\Delta E = E_2 - E_1 = Q - W_T$$

Differentiate this eqn over time

$$\frac{DE}{Dt} = \dot{E} = \underbrace{\frac{\delta Q}{\delta t}}_{\dot{Q}} - \underbrace{\frac{\delta W_T}{\delta t}}_{\dot{W}_T}$$

\dot{Q} = heat transfer rate

\dot{W}_T = total power

$$RTT : \frac{DB}{Dt} = \frac{\partial}{\partial t} \iint_{C,T} g \cdot b dA + \iint_{C,S} g \cdot b (\vec{V} \cdot \hat{n}) dA$$

$$\text{insert } E \text{ to } B \quad b = \frac{B}{m} = e$$

$$e = \underline{e_k} + \underline{e_p} + \underline{U} = \frac{V^2}{2} + gz + U$$

$$E = E_k + E_p + U = \frac{mv^2}{2} + mgz + U$$

$$\frac{DE}{Dt} = \dot{Q} - \dot{W}_T = \cancel{\frac{\partial}{\partial t}} \iint_{C,T} g \left(\frac{V^2}{2} + gz + u \right) dA$$

$$+ \iint_{C,S} g \left(\frac{V^2}{2} + gz + u \right) (\vec{V} \cdot \hat{n}) dA$$

surface forces \rightarrow Flow power (\dot{W}_P): due to the surface forces created by the surrounding fluid.

\rightarrow Everything else (\dot{W}) turbine, shaft, propeller

$$\vec{F}_P = \iint (-p \cdot \hat{n}) \cdot dA$$

$$\dot{\omega}_F = \frac{d\omega_F}{dt} \stackrel{\text{c.s.}}{=} -\frac{d}{dt} (F_p \cdot \alpha) \\ = -F_p \cdot \frac{d\alpha}{dt}$$

$$\dot{\omega}_F = \iint_{C.S.} p(\vec{V} \cdot \hat{n}) dA$$

$$\dot{\omega}_T = \dot{\omega} + \iint_{C.S.} p(\vec{V} \cdot \hat{n}) dA$$

$\dot{\omega}_F$

$$\frac{DE}{Dt} = Q \circlearrowleft \left[\dot{\omega} + \iint_{C.S.} p(\vec{V} \cdot \hat{n}) dA \right] =$$

$$\frac{\partial}{\partial t} \iiint_{C.V.} g \cdot e dV + \iint_{E.S.} g \cdot e (\vec{V} \cdot \hat{n}) dA$$

$$Q - W = \frac{\partial}{\partial t} \iiint_{C.T.} g \left(\frac{v^2}{2} + gz + u \right) dV$$

$$+ \iint_{C.S.} g \left(\frac{v^2}{2} + gz + u + \underbrace{\frac{P}{g}}_{h} (\vec{V} \cdot \hat{n}) \right) dA$$

$\vec{h} \rightarrow \text{enthalpy}$

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t} \iiint_{C.T.} g \left(\frac{v^2}{2} + gz + h \right) dV$$

$$+ \iint_{C.S.} g \left(\frac{v^2}{2} + gz + h \right) (\vec{V} \cdot \hat{n}) dA$$

Special Cases for Conservation of Energy:

I) Steady ($\frac{\partial \dots}{\partial t} = 0$) $\frac{\partial}{\partial t} \iiint_{C.T.} g \left(\frac{v^2}{2} + gz + u \right) dV = 0$

$$\dot{Q} - \dot{W} = \iint_{C.S.} g \left(\frac{v^2}{2} + gz + h \right) (\vec{V} \cdot \hat{n}) dA$$

II) steady + uniform flow + one inlet/one outlet

$$\dot{Q} - \dot{W} = \underbrace{\rho_e \left(\frac{V^2}{2} + g z + h \right)}_e N_{ne} A_e - \underbrace{\rho_i \left(\frac{V^2}{2} + g z + h \right)}_i V_n A_i$$

$$\dot{Q} - \dot{W} = \underbrace{\dot{m}_e \left(\frac{V^2}{2} + g z + h \right)}_e - \underbrace{\dot{m}_i \left(\frac{V^2}{2} + g z + h \right)}_i$$

C.O.Mass $\underbrace{\dot{m}_e V_{ne} A_e}_{\dot{m}_e} = \underbrace{\dot{m}_i V_n A_i}_{\dot{m}_i}$

$$\dot{Q} - \dot{W} = \dot{m} \left[\frac{V^2}{2} + g z + h \right]_e - \left[\frac{V^2}{2} + g z + h \right]_i$$

$\uparrow \frac{J}{s}$
~~watt~~
 $\frac{kg}{s}$

$$q - w = \left(\frac{V^2}{2} + g z + h \right)_e - \left(\frac{V^2}{2} + g z + h \right)_i$$

$\uparrow \frac{J}{kg}$

$q = \text{heat transfer / mass}$
 $w = \text{work / mass}$

$\frac{J}{kg}$

$$q - w = \left(\frac{V^2}{2} + gz + \frac{P}{\rho g} + u_e \right) e - \left(\frac{V^2}{2} + gz + \frac{P}{\rho g} + u_i \right)_i$$

$$\underbrace{q - u_e + u_i - w}_{\text{loss } l} = \left(\frac{V^2}{2} + gz + \frac{P}{\rho g} \right)_e - \left(\frac{V^2}{2} + gz + \frac{P}{\rho g} \right)_i$$

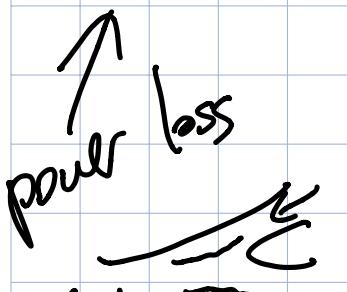
$$l - w = \left(\frac{V^2}{2} + gz + \frac{P}{\rho g} \right)_e - \left(\frac{V^2}{2} + gz + \frac{P}{\rho g} \right)_i$$

negloss
per mass

$$l \leq 0$$

↑ only for
one inlet
one outlet

$$L - W = \sum_{\text{exits}} \dot{m}_e \left(\frac{V^2}{2} + gz + \frac{P}{\rho g} \right)_e$$



$$- \sum_{\text{inlets}} \dot{m}_i \left(\frac{V^2}{2} + gz + \frac{P}{\rho g} \right)_i$$

$$l \rightarrow \frac{J}{kg} \cdot \frac{kg}{s} = \frac{J}{s} = \text{Watt}$$

Water is flowing in a 10 cm, 1 m long pipe.

The pressure upstream is 70 kPa, while the pressure downstream is 65 kPa.

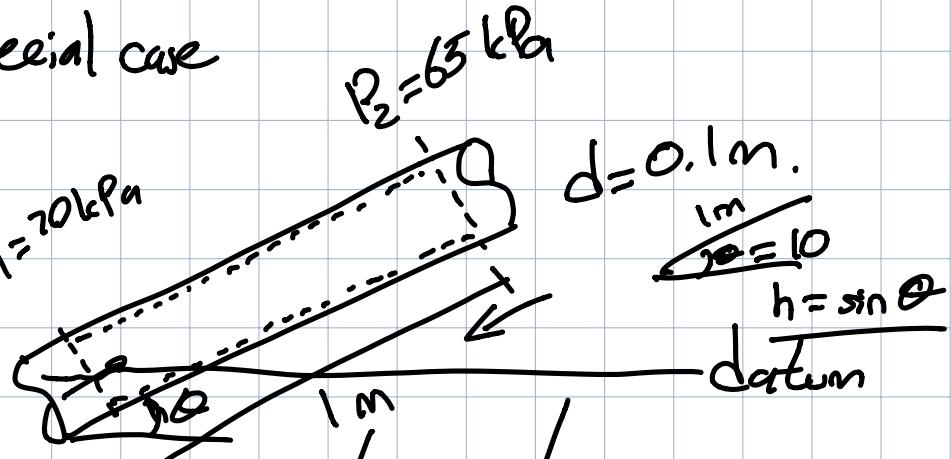
- what is energy lost per unit mass if $\theta = 10^\circ$
- what is energy lost per unit mass if $\theta = 45^\circ$
- What is the maximum angle the pipe can be tilted before the flow stops?

assumptions / special case

- steady

- const. density

- uniform flow



$$\text{C.O. Mass} \implies V_1 A_1 = V_2 A_2 \implies \underline{V_1 = V_2}$$

$$\text{C.O.E} \implies P_1 - \cancel{\psi_1} = \left(\frac{V_1^2}{2} + g z_1 + \frac{P_1}{\rho} \right)_1 - \left(\frac{V_2^2}{2} + g z_2 + \frac{P_2}{\rho} \right)_2$$

$$P = \left(g \sin \theta + \frac{65,000}{1000} \right) - \left(\frac{70,000}{1000} \right)$$

$$P = 9.81 \sin \theta - 5$$

a) take $\theta = 10^\circ \Rightarrow P = -3.3 \frac{\text{J}}{\text{kg}}$

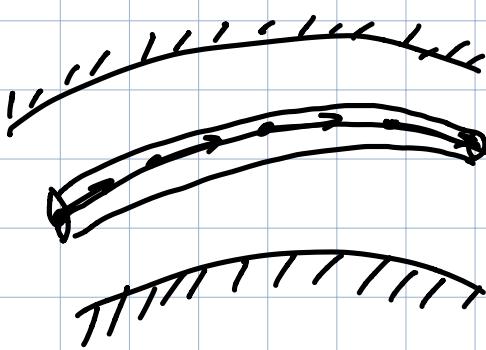
b) take $\theta = 45^\circ \Rightarrow P = +1.94 \frac{\text{J}}{\text{kg}}$

c) static $P = 0$

$$9.81 \sin \theta = 5 \quad \sin \theta = \frac{5}{9.81} \Rightarrow$$

$$\theta = 30.6^\circ$$

Bernoulli's equation:



one inlet / one outlet

Streamline: a line/curve tangent to the flow velocity.

* streamlines do not cross each other.

$$\left(\frac{V^2}{2} + gz + \frac{P}{\rho g} \right)_e - \left(\frac{V^2}{2} + gz + \frac{P}{\rho g} \right)_i = f - w$$

assume - ρ is constant

- no work

- no loss

$$\left(\frac{V^2}{2} + gz + \frac{P}{\rho} \right)_e - \left(\frac{V^2}{2} + gz + \frac{P}{\rho} \right)_i = 0$$

$$\left(\frac{V^2}{2} + gz + \frac{P}{\rho} \right)_e = \left(\frac{V^2}{2} + gz + \frac{P}{\rho} \right)_i = C$$

$$\boxed{\frac{V^2}{2} + gz + \frac{P}{\rho} = C}$$

along any
one
streamline

C varies from streamline to streamline

- only applicable for any 2 points on a
single streamline

- no work

- no loss

- steady flow

- $\rho = \text{const}$

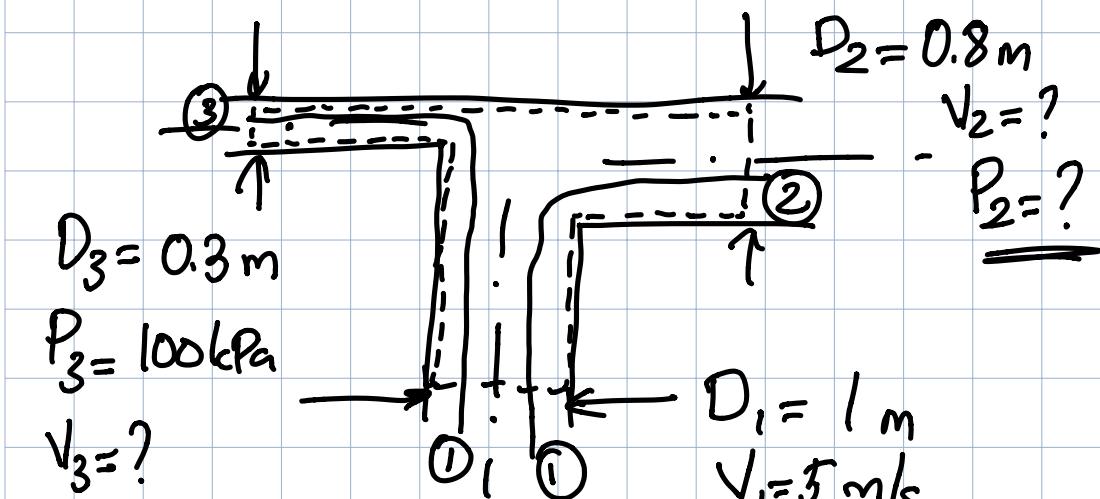
- inviscid flow

divide both sides by g (gravity)

$$\frac{V^2}{2g} + z + \frac{P}{\rho g} = \frac{C}{g} = D$$

↑ elevation head
 velocity head ↓ pressure head

Water flows through a horizontal tee as shown in the figure below. Inlet velocity and pressure are 5 m/s and 200 kPa, respectively. Determine the pressure at the exit section of the tee marked as ② if the pressure at exit section ③ is 100 kPa.



Assumptions/Special cases $P_i = 200 \text{ kPa}$

- steady
- constant density

C.O. Mass

$$\sum V_i A_i = \sum V_o A_o$$

- Uniform flow.

inlet " " exit "

- inviscid

$$V_1 A_1 = V_2 A_2 + V_3 A_3$$

- no loss

$$(5) \left(\frac{\pi}{4} (1)^2 \right) = V_2 \left(\frac{\pi}{4} (0.8)^2 \right) + V_3 \left(\frac{\pi}{4} (0.3)^2 \right)$$

- no work

$$5 = 0.64 V_2 + 0.09 V_3$$

Bernoulli's eqn $① \rightarrow ③$

$$\frac{P_1}{g} + \frac{V_1^2}{2} + g z_1 = \frac{P_3}{g} + \frac{V_3^2}{2} + g z_3$$

$$\frac{200,000}{(1000)} + \frac{(5)^2}{2} = \frac{100,000}{(1000)} + \frac{V_3^2}{2}$$

$$200 + (12.5) = 100 + \frac{V_3^2}{2}$$

$$\frac{V_3^2}{2} = 12.5$$

$$V_3^2 = 225 \Rightarrow V_3 = 15 \text{ m/s}$$

$$5 = (0.64)(V_2) + 0.09(15)$$

$$\Rightarrow V_2 = 5.7 \text{ m/s}$$

Bernoulli's eqn. $① \rightarrow ②$

$$\frac{P_1}{g} + \frac{V_1^2}{2} + g\beta_1 = \frac{P_2}{g} + \frac{V_2^2}{2} + g\beta_2$$

$$\frac{\cancel{200,000}}{(1000)} + \frac{(5)^2}{2} = \frac{P_2}{(1000)} + \frac{(5.7)^2}{2}$$

$$212.5 = \frac{P_2}{1000} + 16.25$$

$$P_2 = 196,250 P_1$$

$$\boxed{P_2 = 196.25 \text{ kPa}}$$

Water flows through a horizontal tee as shown in the figure below. Inlet velocity and pressure are 5 m/s and 300 kPa, respectively, while the exit (2) and (3) information are shown in the figure below. Find the power lost in this tee.



$$l_3 = 100 \text{ cm} \quad \xrightarrow{\text{1.1.1.1}} \quad l_1 = 5 \text{ m/s} \\ V_3 = 15 \text{ m/s} \quad \textcircled{1}, \textcircled{1} \quad P_1 = 300 \text{ kPa}$$

Assumptions / Special Cases

- steady
- constant density
- uniform flow

C.O. Mass

$$\sum_{\text{inlets}} V_{in} A_{in} = \sum_{\text{exits}} V_e A_e$$

$$V_1 A_1 = V_2 A_2 + V_3 A_3$$

$$(5) \left(\frac{\pi}{4} \right) (1)^2 = V_2 \cdot \frac{\pi}{4} (0.8)^2 + (15) \frac{\pi}{4} (0.3)^2$$

$$5 = 0.64 V_2 + 1.35$$

$$V_2 = \frac{5 - 1.35}{0.64} = 5.7 \text{ m/s}$$

$$\dot{L} - \dot{W} = \sum_{\text{exits}} \dot{m}_e \left(\frac{V^2}{2} + gz + \frac{P}{\rho} \right)_e$$

$$- \sum_{\text{inlets}} \dot{m}_i \left(\frac{V^2}{2} + gz + \frac{P}{\rho} \right)_i$$

$$\dot{L} = \dot{m}_2 \left(\frac{V_2^2}{2} + gz + \frac{P}{\rho} \right) + \dot{m}_3 \left(\frac{V_3^2}{2} + gz + \frac{P}{\rho} \right)$$

$$-\dot{m}_1 \left(\frac{V_1^2}{2} + \gamma P_1 + \frac{\dot{P}}{g} \right)$$

$$\dot{m}_1 = g V_1 A_1 = (1000)(5) \left(\frac{\pi}{4}\right) (1)^2 = 3927 \frac{\text{kg}}{\text{s}}$$

$$\dot{m}_2 = g V_2 A_2 = (1000)(5.7) \left(\frac{\pi}{4}\right) (0.8)^2 = 2865 \frac{\text{kg}}{\text{s}}$$

$$\dot{m}_3 = g V_3 A_3 = (1000)(15) \left(\frac{\pi}{4}\right) (0.3)^2 = 1060 \frac{\text{kg}}{\text{s}}$$

$$\dot{Q} = 2865 \left(\frac{209,000}{1000} + \frac{(5.7)^2}{2} \right) \leftarrow ②$$

$$+ 1060 \left(\frac{100,000}{1000} + \frac{(15)^2}{2} \right) \leftarrow ③$$

$$- 3927 \left(\frac{300,000}{1000} + \frac{(5)^2}{2} \right) \leftarrow ①$$

$$\dot{Q} = -382,395 \text{ Watts}$$

$$\boxed{\dot{W} = -3821.1 \text{ kJ}}$$

λ