

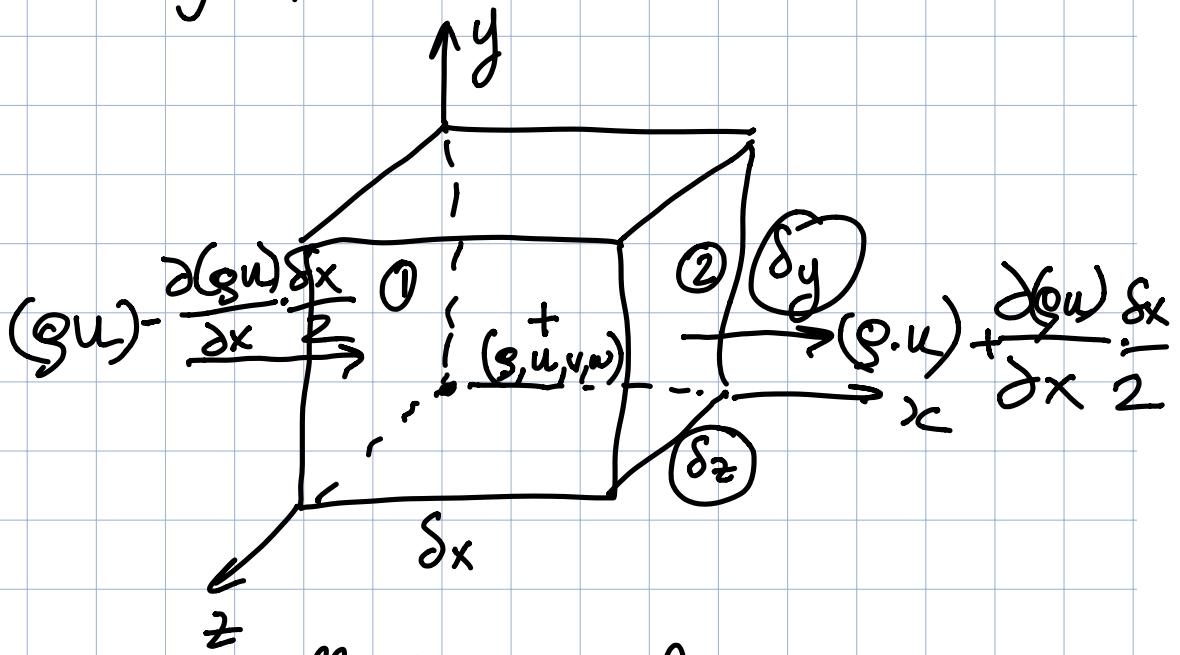
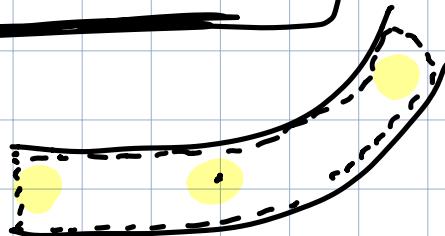
Differential Analysis of Fluid Flow:

Conservation of Mass :

$$\frac{\partial}{\partial t} \iiint_{C.V} \rho dV + \iint_{C.S} \rho (\vec{V} \cdot \hat{n}) dA = 0$$

shrink C.V

$$V = (\delta_x)(\delta_y)(\delta_z)$$



as C.V is small, density of constant.

$$\frac{\partial}{\partial t} \iiint_{C.V} \rho dV = \frac{\partial \rho}{\partial t} \iiint_{C.V} dV = \left[\frac{\partial \rho}{\partial t} \cdot V \right]$$

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$$\int \underline{dx} \rightarrow \underline{x}$$

$$\iint \underline{dA} \rightarrow \iint dxdy = x \cdot y = \underline{A}$$

$$\iiint \underline{dT} \rightarrow \iiint dx dy dz = x \cdot y \cdot z = \underline{T}$$

$$\iint_{C.S} g(\vec{V} \cdot \hat{n}) dA = \iint g(-u) dA + \iint g(u) dA$$

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$$= - \underbrace{(g.u)}_1 A_1 + \underbrace{(g.u)}_2 A_2$$

$$= - \left[\underbrace{(g.u)}_1 - \frac{\partial(gu)}{\partial x} \cdot \frac{\delta_x}{2} \right] \cdot \underline{\delta_y \delta_z}$$

$(g.u)_1$

$$+ \left[\underbrace{(g.u)}_1 + \frac{\partial(gu)}{\partial x} \cdot \frac{\delta_x}{2} \right] \cdot \underline{\delta_y \delta_z}$$

$$\begin{aligned}
 &= \left[\frac{\partial(gu)}{\partial x} \cdot \frac{\delta_x}{2} \right] \delta_y \delta_z + \left[\frac{\partial(gu)}{\partial x} \cdot \frac{\delta_x}{2} \right] \delta_y \delta_z \\
 &= \cancel{\left[\frac{\partial(gu)}{\partial x} \cdot \frac{\delta_x}{2} \right]} \cdot \delta_y \delta_z = \boxed{\frac{\partial(gu)}{\partial x} \cdot \nabla}
 \end{aligned}$$

$$\frac{\partial(gv)}{\partial y} \cdot \nabla$$

$\rightarrow y$ direction

$$\frac{\partial(gw)}{\partial z} \cdot \nabla$$

$\rightarrow z$ direction

$$\begin{aligned}
 &\frac{\partial g}{\partial t} \cdot \nabla + \frac{\partial(gu)}{\partial x} \nabla + \frac{\partial(gv)}{\partial y} \nabla + \frac{\partial(gw)}{\partial z} \nabla = 0 \\
 &\underbrace{\frac{\partial g}{\partial t} \cdot \nabla}_{1^{\text{st}} \text{ term}} \quad \underbrace{\frac{\partial(gu)}{\partial x} \nabla + \frac{\partial(gv)}{\partial y} \nabla + \frac{\partial(gw)}{\partial z} \nabla}_{2^{\text{nd}} \text{ term}}
 \end{aligned}$$

$$\nabla \cdot \left[\frac{\partial g}{\partial t} + \frac{\partial(gu)}{\partial x} + \frac{\partial(gv)}{\partial y} + \frac{\partial(gw)}{\partial z} \right] = 0$$

$\nwarrow A \cdot B = 0$

$$\frac{\partial g}{\partial t} + \frac{\partial(gu)}{\partial x} + \frac{\partial(gr)}{\partial y} + \frac{\partial(gw)}{\partial z} = 0$$

Special Cases of Continuity:

I) steady flows: $\Rightarrow \frac{\partial g}{\partial t} = 0$

$$\cancel{\frac{\partial g}{\partial t}} + \frac{\partial(gu)}{\partial x} + \frac{\partial(gr)}{\partial y} + \frac{\partial(gw)}{\partial z} = 0$$

$$\frac{\partial(gu)}{\partial x} + \frac{\partial(gr)}{\partial y} + \frac{\partial(gw)}{\partial z} = 0$$

II) Incompressible flow: $g = \text{const.}$

$$\frac{\partial(gw)}{\partial x} + \frac{\partial(gr)}{\partial y} + \frac{\partial(gw)}{\partial z} = 0$$

$$g\left(\frac{\partial u}{\partial x}\right) + g\left(\frac{\partial v}{\partial y}\right) + g\left(\frac{\partial w}{\partial z}\right) = 0$$

$$g \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] = 0$$

$$\begin{matrix} \nearrow \partial x \\ A \cdot B = 0 \end{matrix} \quad \begin{matrix} \partial y \\ \downarrow \partial z \end{matrix} \quad \begin{matrix} \nearrow \partial z \\ \downarrow \end{matrix}$$

either $A=0$ or
 $B=0$

Example: For a 2-D flow $u=2x$

Determine a possible y component of velocity,
for an incompressible flow $v=?$

incompressible $\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\underbrace{\frac{\partial 2x}{\partial x}}_{=} + \frac{\partial v}{\partial y} = 0$$

$$2 + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial y} = -2$$

$$v = -2y + f(x)$$

$$\frac{dv(y)}{y} = -2 \Rightarrow v = -2y + C$$

$$\frac{\partial (-2y + f(x))}{\partial y} = -2$$

$$V = u \hat{i} + v \hat{j} + w \hat{k}$$

$$\vec{V} = 2x\hat{i} + (-2y + f(x))\hat{j}$$

Stream function (ψ)

consider a 2-D incompressible flow in Cartesian coordinates

2-D $\Rightarrow u, v$ present $w=0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cancel{\frac{\partial w}{\partial z}} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

introduce a stream function ($\psi(x, y, t)$)

$$u = \frac{\partial \psi}{\partial y}$$

and

$$v = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial y} \right) = \frac{\partial^2 \psi}{\partial x \partial y}$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y} \left(-\frac{\partial \psi}{\partial x} \right) = -\frac{\partial^2 \psi}{\partial y \partial x}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial^2 \psi}{\partial x \partial y} + \left(-\frac{\partial^2 \psi}{\partial y \partial x} \right) ? = 0$$

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$$V = e^x + \ln y$$

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